One Pile Nim with Arbitrary Move Function

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Abstract

The purpose of this paper is to solve a class of combinatorial games consisting of one-pile counter pickup games for which the maximum number of counters that can be removed on each successive move changes during the play of the game.

Two players alternate removing a positive number of counters from the pile. An ordered pair (N, x) of positive integers is called a position. The number N represents the size of the pile of counters, and x represents the greatest number of counters that can be removed on the next move. A function $f : Z^+ \to Z^+$ is given which determines the maximum size of the next move in terms of the current move size. Thus a move in a game is an ordered pair of positions $(N, x) \mapsto (N - k, f(k))$, where $1 \le k \le \min(N, x)$. The game ends when there are no counters left, and the winner is the last player to move in a game. In this paper we will consider $f : Z^+ \to Z^+$ to be completely arbitrary. This paper extends a previous paper by the authors [5], which in turn extended two other papers, [3] and [6]. The paper by Epp, Ferguson [3] assumed f is non-decreasing, and the paper by Schwenk [6] assumed f is non-decreasing and $f(n) \ge n$. Our previous paper [5] assumed more general conditions of f including as a special case all $f : Z^+ \to Z^+$ that satisfy $f(n + 1) - f(n) \ge -1$. The main theorem of this paper will also allow the information concerning the strategy of a game to be stored very efficiently, and our paper [5] is a subcase of this paper. We now proceed to develop the theory.

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