Orthogonal polynomials represented by CW-spheres

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Abstract

Given a sequence $\{Q_n(x)\}_{n=0}^{\infty}$ of symmetric orthogonal polynomials, defined by a recurrence formula $Q_n(x) = \nu_n \cdot x \cdot Q_{n-1}(x) - (\nu_n - 1) \cdot Q_{n-2}(x)$ with integer ν_i's satisfying $\nu_i \geq 2$, we construct a sequence of nested Eulerian posets whose *ce*-index is a non-commutative generalization of these polynomials. Using spherical shellings and direct calculations of the *cd*-coefficients of the associated Eulerian posets we obtain two new proofs for a bound on the true interval of orthogonality of $\{Q_n(x)\}_{n=0}^{\infty}$. Either argument can replace the use of the theory of chain sequences. Our *cd*-index calculations allow to represent the orthogonal polynomials as an explicit positive combination of terms of the form $x^{n-2r}(x^2-1)^r$. Both proofs may be extended to the case the ν_i's are not integers and the second proof is still valid when only $\nu_i > 1$ s required. The construction provides a new "limited testing ground" for Stanley's non-negativity conjecture for Gorenstein* posets, and suggests the existence of strong links between the theory of orthogonal polynomials and flag-enumeration in Eulerian posets.

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