Weyl-Heisenberg Frame Wavelets with Basic Supports

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Preprint no. 2005-06
August 24, 2005

Abstract

Let a, b be two fixed non-zero constants. A measurable set $E \subset \mathbb{R}$ is called a Weyl-Heisenberg frame set for (a,b) if the function $g = \chi_E$ generates a Weyl-Heisenberg frame for $L^2(\mathbb{R})$ under modulates by b and translates by a, i.e., $\{e^{imbt}g(t-na)\}_{m,n\in\mathbb{Z}}$ is a frame for $L^2(\mathbb{R})$. It is an open question on how to characterize all frame sets for a given pair (a,b) in general. In the case that $a = 2\pi$ and $b = 1$, a result due to Casazza and Kalton shows that the condition that the set $F = \bigcup_{j=1}^{k}([0,2\pi) + 2n_j\pi)$ (where $\{n_1 < n_2 < \cdots < n_k\}$ are integers) is a Weyl-Heisenberg frame set for $(2\pi, 1)$ is equivalent to the condition that the polynomial $f(z) = \sum_{j=1}^{k} z^{n_j}$ does not have any unit roots in the complex plane. In this paper, we show that this result can be generalized to a class of more general measurable sets (called basic support sets) and to set theoretical functions and continuous functions defined on such sets.

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