Total Curvature, Ropelength and Crossing Number of Thick Knots

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Abstract

We first study the minimum total curvature of a knot when it is embedded on the cubic lattice. Let $\mathcal{K}$ be a knot or link with a lattice embedding of minimum total curvature $\tau(\mathcal{K})$ among all possible lattice embeddings of $\mathcal{K}$. We show that there exist positive constants c_1 and c_2 such that $c_1\sqrt{Cr(\mathcal{K})} \leq \tau(\mathcal{K}) \leq c_2 Cr(\mathcal{K})$ for any knot type $\mathcal{K}$. Furthermore we show that the powers of $Cr(\mathcal{K})$ in the above inequalities are sharp hence cannot be improved in general. Our results and observations show that lattice embeddings with minimum total curvature are quite different from those with minimum or near minimum lattice embedding length. In addition, we discuss the relationship between minimal total curvature and minimal ropelength for a given knot type. At the end of the paper, we study the total curvatures of smooth thick knots and show that there are some essential differences between the total curvatures of smooth thick knots and lattice knots.

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