

TUTTE POLYNOMIALS OF TENSOR
PRODUCTS OF SIGNED GRAPHS AND THEIR
APPLICATIONS IN KNOT THEORY

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Abstract

It is well-known that the Jones polynomial of an alternating knot is closely related to the Tutte polynomial of a special graph obtained from a regular projection of the knot. Relying on the results of Bollobás and Riordan, we introduce a generalization of Kauffman's Tutte polynomial of signed graphs for which describing the effect of taking a signed tensor product of signed graphs is very simple. We show that this Tutte polynomial of a signed tensor product of signed graphs may be expressed in terms of the Tutte polynomials of the original signed graphs by using a simple substitution rule. Our result enables us to compute the Jones polynomials of some large non-alternating knots. The combinatorics used to prove our main result is similar to Tutte's original way of counting "activities" and specializes to a new, perhaps simpler proof of the known formulas for the ordinary Tutte polynomial of the tensor product of unsigned graphs or matroids.

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