

DELANNOY ORTHANTS OF LEGENDRE
POLYTOPES

Gábor Hetyei

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Abstract

We construct an n -dimensional polytope whose boundary complex is compressed and whose face numbers for any pulling triangulation are the coefficients of the powers of $(x - 1)/2$ in the n -th Legendre polynomial. We show that the non-central Delannoy numbers count all faces in the lexicographic pulling triangulation that contain a point in a given open generalized orthant. We thus provide a geometric interpretation of a relation between the central Delannoy numbers and Legendre polynomials, observed over 50 years ago. The polytopes we construct are closely related to the root polytopes introduced by Gelfand, Graev, and Postnikov.

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