

SHIFTED JACOBI POLYNOMIALS AND DELANNOY NUMBERS

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Abstract

We express a weighted generalization of the Delannoy numbers in terms of shifted Jacobi polynomials. A specialization of our formulas extends a relation between the central Delannoy numbers and Legendre polynomials, observed over 50 years ago, to all Delannoy numbers and certain Jacobi polynomials. Another specialization provides a weighted lattice path enumeration model for shifted Jacobi polynomials and allows the presentation of a combinatorial, non-inductive proof of the orthogonality of Jacobi polynomials with natural number parameters. The proof relates the orthogonality of these polynomials to the orthogonality of (generalized) Laguerre polynomials, as they arise in the theory of rook polynomials. We also find finite orthogonal polynomial sequences of Jacobi polynomials with negative integer parameters and expressions for a weighted generalization of the Schröder numbers in terms of the Jacobi polynomials.

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