# The Linearity of the Ropelenghts of Conway Algebraic Knots in Terms of Their Crossing Numbers

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**Abstract**

For a knot or link $\mathcal{K}$, let $L(\mathcal{K})$ denote the ropelength of $\mathcal{K}$ and let $Cr(\mathcal{K})$ denote the crossing number of $\mathcal{K}$. An important problem in geometric knot theory concerns the relationship between $L(\mathcal{K})$ and $Cr(\mathcal{K})$ (or intuitively, the relationship between the length of a rope needed to tie a particular knot and the complexity of the knot). We show that there exists a constant $a > 0$ such that if a knot $\mathcal{K}$ allows a special knot diagram $D$ (called Conway algebraic knot diagram) with $n$ crossings, then $L(\mathcal{K}) \leq a \cdot n$. Furthermore, if $D$ is alternating (but not necessarily reduced and in fact $\mathcal{K}$ may not have a minimal alternating diagram that is algebraic), then $L(\mathcal{K}) \leq a \cdot Cr(\mathcal{K})$. The approach used here can be applied to a larger class of knots, namely those formed by replacing single crossings in a Conway algebraic knot diagram by tangles whose crossing number is bounded by a constant. Interestingly, it has been shown by the same authors that the Jones polynomials of these knots can be computed in polynomial time.

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