The Growth of the Mean Average Crossing Number of Equilateral polygons in Confinement

J. Arsuaga, B. Borgo,
Y. Diao and R. Scharein

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Abstract

The physical and biological properties of collapsed long polymer chains as well as of highly condensed biopolymers (such as DNA in all organisms) are known to be determined, at least in part, by their topological and geometrical properties. With this purpose of characterizing the topological properties of such condensed systems equilateral random polygons restricted to confined volumes are often used. However very few analytical results are known. In this paper, we investigate the effect of volume confinement on the mean average crossing number (ACN) of equilateral random polygons. The mean ACN of knots and links under confinement provides a simple alternative measurement for the topological complexity of knots and links in the statistical sense. For an equilateral random polygon of n segments without any volume confinement constrain, it is known that its mean ACN $\langle \text{ACN} \rangle$ is of the order $\frac{3}{16} n \ln n + O(n)$. Here we model the confining volume as a simple sphere of radius R. We provide an analytical argument that shows that $\langle \text{ACN} \rangle$ of an equilateral random polygon of n segments under extreme confinement (meaning $R << n$) grows as $O(n^2)$. We propose to model the growth of $\langle \text{ACN} \rangle$ as $a(R)n^2 + b(R)n \ln(n)$ under a less extreme confinement condition, where $a(R)$ and $b(R)$ are functions of R with R being the radius of the confining sphere. Computer simulations performed show a fairly good fit using this model.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223