Intrinsic s-elementary Parseval Frame Multiwavelets in $L^2(\mathbb{R}^d)$

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Preprint no. 2009-08

Abstract

A countable family $\{x_j\}, j \in J$, in a separable Hilbert space H, is a Parseval frame for H if $\|f\|^2 = \sum_{j\in J} |\langle f, x_j\rangle|^2$ holds for all $f \in H$. In the case that $H = L^2(\mathbb{R}^d)$ and the affine system $\{D_A^j T_k \psi_i | j \in \mathbb{Z}, k \in \mathbb{Z}^d, 1 \leq i \leq n\}$ obtained from a finite subset $\Psi = \{\psi_1, \psi_2, \cdots, \psi_n\}$ of $L^2(\mathbb{R}^d)$ is a Parseval frame, Ψ is called an A-dilation Parseval frame multiwavelet (of length n). Here A stands for a $d \times d$ expansive matrix, and T, D_A are the translation and A-dilation unitary operators acting on $L^2(\mathbb{R}^d)$ defined by $(T^\ell f)(\mathbf{t}) = \mathbf{f}(\mathbf{t} - \ell)$, $(D_A f)(\mathbf{t}) = |\det \mathbf{A}|^{\frac{1}{2}} \mathbf{f}(\mathbf{A}\mathbf{t})$, $\forall f \in L^2(\mathbb{R}^d)$, $\ell \in \mathbb{Z}^d$, $\mathbf{t} \in \mathbb{R}^{\mathbf{d}}$. In the special case that there exist disjoint measurable sets $\{E_1, E_2, ..., E_n\}$ such that $\widehat{\psi}_i = \frac{1}{(2\pi)^{d/2}} \chi_{E_i}$ for each i, Ψ is called an A-dilation s-elementary Parseval frame multiwavelet. A measurable set E is called a frame multiwavelet set of multiplicity m (under A-dilation) if E can be written as a disjoint union of measurable sets $\{E_1, E_2, ..., E_m\}$ such that $\widehat{\psi}_i = \frac{1}{(2\pi)^{d/2}} \chi_{E_i}$ defines a Parseval frame multiwavelet $\Psi = \{\psi_1, \psi_2, \cdots, \psi_n\}$, and that this cannot be done for any integer less than m. An A-dilation s-elementary Parseval frame multiwavelet with length m that is defined on a frame multiwavelet set of multiplicity m is said to be intrinsic. It is known that single A-dilation wavelets exist in $L^2(\mathbb{R}^d)$ for any expansive matrix A. In this paper, we show that for any $d \times d$ expansive matrix A and any given $m \in \mathbb{N}$,the family of intrinsic A-dilation s-elementary Parseval frame multiwavelet with length m is not empty, and is path-connected under the norm topology of $(L^2(\mathbb{R}^d))^m$. The same result holds for the family of all intrinsic A-dilation s-elementary Parseval frame multiwavelets of length m.

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