Modeling of Chromosome Intermingling by Partially Overlapping Uniform Random Polygons

T. Blackstone, R. Scharein, B. Borgo,
R. Varela, Y. Diao and J. Arsuaga

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Abstract

During the early phase of the cell cycle the eukaryotic genome is organized into chromosome territories. The geometry of the interface between any two chromosomes remains a matter of debate and may have important functional consequences. The Interchromosomal Network model (introduced by Branco and Pombo) proposes that territories intermingle along their periphery. In order to partially quantify this concept we here investigate the probability that two chromosomes form an unsplittable link. We use the uniform random polygon as a crude model for chromosome territories and we model the interchromosomal network as the common spatial region of two overlapping uniform random polygons. This simple model allows us to derive some rigorous mathematical results as well as to perform computer simulations easily. We find that the probability that one uniform random polygon of length n that partially overlaps a fixed polygon is bounded below by $1 - O(\frac{1}{\sqrt{n}})$. We use numerical simulations to estimate the dependence of the linking probability of two uniform random polygons on the amount of overlapping. We propose that this dependence relation may be modeled as $f(\varepsilon, m, n) = 1 - \frac{a(\epsilon)}{b(\epsilon)\sqrt{mn}+c(\epsilon)}$. Numerical evidence shows that this model works well when ϵ is relatively large ($\varepsilon \geq .5$). We then use these results to model the data published by Branco and Pombo and observe that for the amount of overlapping observed experimentally the URPs have a non-zero probability of forming an unsplittable link.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223