On s-elementary Super Frame Wavelets and their Path-connectedness

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Abstract

A super wavelet of length n is an n-tuple $(\psi_1, \psi_2, ..., \psi_n)$ in the product space $\prod_{j=1}^n L^2(\mathbb{R})$, such that the *coordinated dilates* of all its *coordinated translates* form an orthonormal basis for $\prod_{j=1}^n L^2(\mathbb{R})$. This concept is generalized to the so-called super frame wavelets, super tight frame wavelets and super normalized tight frame wavelets (or super Parseval frame wavelets), namely an n-tuple $(\eta_1, \eta_2, ..., \eta_n)$ in $\prod_{j=1}^n L^2(\mathbb{R})$ such that the coordinated dilates of all its coordinated translates form a frame, a tight frame, or a normalized tight frame for $\prod_{j=1}^n L^2(\mathbb{R})$. In this paper, we study the super frame wavelets and the super tight frame wavelets whose Fourier transforms are defined by set theoretical functions (called s-elementary frame wavelets). An n-tuple of sets $(E_1, E_2, ..., E_m)$ is said to be τ-disjoint if the E_j's are pair-wise disjoint under the 2π-translations. We prove that a τ-disjoint n-tuple $(E_1, E_2, ..., E_m)$ of frame sets (i.e., η_j defined by $\widehat{\eta_j} = \frac{1}{\sqrt{2\pi}}\chi_{E_j}$ is a frame wavelet for $L^2(\mathbb{R})$ for each j) lead to a super frame wavelet $(\eta_1, \eta_2, ..., \eta_m)$ for $\prod_{j=1}^n L^2(\mathbb{R})$ where $\widehat{\eta_j} = \frac{1}{\sqrt{2\pi}}\chi_{E_j}$. In the case of super tight frame wavelets, we prove that $(\eta_1, \eta_2, ..., \eta_m)$, defined by $\widehat{\eta_j} = \frac{1}{\sqrt{2\pi}}\chi_{E_j}$, is a super tight frame wavelet for $\prod_{1\le j\le m} L^2(\mathbb{R})$ with frame bound k_0 if and only if each η_j is a tight frame wavelet for $L^2(\mathbb{R})$ with frame bound k_0 and that $(E_1, E_2, ..., E_m)$ is τ-disjoint. Denote the set of all τ-disjoint s-elementary super frame wavelets for $\prod_{1\le j\le m} L^2(\mathbb{R})$ by $\mathfrak{S}(m)$ and the set of all s-elementary super tight frame wavelets (with the same frame bound k_0) for $\prod_{1\le j\le m} L^2(\mathbb{R})$ by $\mathfrak{S}^{k_0}(m)$. We further prove that $\mathfrak{S}(m)$ and $\mathfrak{S}^{k_0}(m)$ are both path-connected under the $\prod_{1\le j\le m} L^2(\mathbb{R})$ norm, for any given positive integers m and k_0.

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