

GENERALIZED TCHEBYSHEV TRIANGULATIONS

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Abstract

After fixing a triangulation L of a k -dimensional simplex that has no new vertices on the boundary, we introduce a triangulation operation on all simplicial complexes that replaces every k -face with a copy of L , via a sequence of induced subdivisions. The operation may be performed in many ways, but we show that the face numbers of the subdivided complex depend only on the face numbers of the original complex, in a linear fashion. We use this linear map to define a sequence of polynomials generalizing the Tchebyshev polynomials of the first kind and show, that in many cases, but not all, the resulting polynomials have only real roots, located in the interval $(-1, 1)$. Some analogous results are shown also for generalized Tchebyshev polynomials of the higher kind, defined using summing over links of all original faces of a given dimension in our generalized Tchebyshev triangulations. Generalized Tchebyshev triangulations of the boundary complex of a cross-polytope play a central role in our calculations, and for some of these we verify the validity of a generalized lower bound conjecture by the second author.

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