

SYMMETRY AND CONSTRAINTS IN
HYDRODYNAMICS AND MECHANICAL
LOCOMOTION

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Abstract

This dissertation introduces new models for the locomotion and control of mechanical and hydrodynamic systems that exhibit symmetry and constraints. I introduce the class of unbalanced Chaplygin control systems and analyze two new examples in this class — the Chaplygin beanie and the Chaplygin pendulum. I prove that a single-input control strategy is able to generate locomotion and to control heading and speed. Next, I introduce a new model for the locomotion of articulated rigid bodies in ideal fluids and demonstrate that even without added-mass effects arising from asymmetric body shapes, these swimmers may locomote provided symmetry of the fluid boundary is broken, and I conjecture that an underlying geometric phase governs their motion. I then introduce a family of reduced-order models for the interaction of stationary rigid bodies immersed in inviscid incompressible fluids with vorticity. The rigid bodies impose fluid constraints of four types — velocity, direction, distance, and position. It is found that energy is generally conserved but that linear and angular impulse of the fluid are not. The constraints may dramatically alter the system dynamics. I show that constrained vortex dynamics may yield approximate models for the interaction of moving rigid bodies and vortical fluids. Finally, motivated by both the relevance of principal bundles to locomotion and by the mismatch between the data needed to prove theorems vs. the data available in applications, I elucidate the relationship between three definitions of a principal bundle, introduce a new definition, and demonstrate mutual equivalence of all four.