

SCHWARZ METHODS FOR FOURTH-ORDER PROBLEMS CONTAINING SINGULARITIES

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Abstract

We develop numerical methods for analysis of fourth-order partial differential equations on domains with angular corners. For the finite element analysis of fourth-order partial differential equations, we have to use smoother basis functions whose derivatives are continuous. The approximation space for Isogeometric Analysis (IGA) consists of B-spline basis functions with any desired regularity; however, IGA using single patch encounters difficulties in dealing with boundary value problems on irregular shaped polygonal domains. In this dissertation, we introduce an Implicitly Enriched Galerkin method in which singular basis functions resembling singularities are generated through a special geometric mapping and are combined with flat-top partition of unity (PU) functions. Unlike XFEM, this approach does not have singular integral problems. For the cases where multi-patches are necessary because of complex geometry of the problems, we combine the Implicitly Enriched Galerkin method with Schwarz domain decomposition methods to join two patches along their interface. Thanks to Schwarz methods, we are able to break down the problems to smaller subproblems and are able to use different numerical techniques to solve each subproblem for localized treatment. In order to reduce computational cost, we modify our method by applying Multicolor Schwarz and Supplemental Subdomain methods. Various numerical examples show the efficiency of our proposed method in dealing with fourth-order singular problems with crack singularities and/ or corner singularities.