BUTTON DESIGN FOR MAP OVERLAYS (II)

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ABSTRACT

A new approach to button design has recently been proposed that uses natural segments, instead of "even intervals", on the weight value range in determining the number of tabs that should be incorporated in a button system and the value each button carries. Button systems so designed do not have the problems of redundancy and under-representation that are common to the conventionally designed button systems. Presently limited to the map overlays that involve only two parent-maps, this approach is extended to the case of three parent-maps in this article.

1. INTRODUCTION

A popular application of the button technique in a GIS environment is map overlays for land suitability assessment. Herein these functions keys on a computer screen are coded with weight values that indicate parent-maps' importance to their associations. Users select these values by clicking on the appropriate buttons. The values will then be incorporated into the map overlay process to produce suitability maps.

When designing buttons for map overlays, a person must decide how many tabs should be incorporated and what values they bear. Many have adopted an approach that uses odd numbers of weight values (that is, 3, 5, 7, or 9) with an even interval (see, for example, Whitley et al, 1993; Valent and Young, 1995; Secunda et al, 1998; Klosterman, 1999). Unfortunately, this approach is defective and thus subject to errors. According to a recent study (Xiang and Salmon, 2001), a button system that carries weight values of even increments suffers inevitably from the problems of redundancy and under-representation, which can significantly affect the quality of map overlays. Through an example of land suitability assessment for park site selection, the study showed that some tabs in such a button system are equivalent to one another functionally because they produce overlaid maps that share an identical site ranking. Should the site selection be based solely on the rankings, which is often a common practice in the real world decision-making (Holmes, 1972 and 1973; Kirkwood and Corner, 1993; Srivastava et al, 1995; Barron and Barrett, 1996), the use of any one of these maps will lead to virtually the same decision. The inclusion of these equivalent buttons is therefore redundant. Ironically however, the very same button system is also incomplete for it represents only a partial array of preferential choices. A number of overlaid maps that have unique site rankings are concealed and placed out of a user's sight simply because their corresponding weight values are not included in the button set. Since the use of buttons is a way, and in many cases, the only way, to express his perceptions about maps' importance, a user would have made different site selections if he was presented the complete array of preferential choices. From this perspective, the redundant button system is also inadequate and under-representative.

Fortunately, these two problems can be avoided should a person take an alternative approach, *design with segmentation* (Xiang, 2000; Xiang and Salmon, 2001). Herein, the number of buttons and the value each button carries are both determined by the "natural breaks" (that is, segments) on the weight value range $[0, 1]$, instead of the arbitrary "even intervals". This warrants that every individual button in the system correspond to an overlaid map that bears a unique site ranking and that all the buttons together represent the complete array of unique overlaid maps. The resultant button system is therefore assured free from the two problems. With this approach, Xiang and Salmon developed a button system for the weighting of two parent-maps (2001).

In this article, we extend their work to a more general case of three parent-maps. We chose the case of three instead of a higher number for two reasons. First, the existing button design by Xiang and Salmon becomes inadequate in the three parent-map case. The seemly trivial increment from two to three parent-maps elevates the order of fragmentation on the weight value range and, as we shall demonstrate later, imposes a precision level on the overlaid map that is finer than the existing button system can handle. Secondly, many in the area of multi-criteria evaluation believe, and some have convincingly demonstrated, that the case of three factors (that is, three parent-maps in our case) is the key to an understanding of the multiple-factor case (for example, Churchman and Ackoff, 1953; Keeney and Raiffa, 1976). We share their belief and hope that this study on the more manageable case of three maps will shed lights on the ultimate design for the weighting of multiple maps.

Our investigation shall focus on two pieces of knowledge that are essential to this extension: the total number of segments on the weight value range in the presence of three parent-maps, and their value representations, that is, what value should be used to represent each segment. To set up the stage for discussion, however, we shall first review briefly the phenomenon of weight segmentation in the two parent-maps case (Xiang, 2000; Xiang and Salmon, 2001).

2. WEIGHT SEGMENTATION IN THE TWO PARENT-MAP CASE

Let us begin with the following definitions. c_1 and c_2 are the attributes of the parent-maps P_1 and P_2, respectively. $\{c_{1i}\}$ is an ordered set of land capability values of c_1 ($i =$

$1, 2, 3)$ where $c_{11} = 1$ representing the most capable, $c_{12} = 0.5$ the moderately, and $c_{13} = 0$ the least. $\{c_{2j}\}$ is $\{c_{1i}\}$'s counterpart for c_2 with $c_{21} = 1$ as the most capable, $c_{22} = 0.5$ the moderately, and $c_{23} = 0$ the least. w_1 and w_2 are the attribute weights for c_1 and c_2, respectively, representing their importance to the assessment. Both w_1 and w_2 are defined on the weight value range $[0, 1]$ where a value of 1 represents the highest level of importance and 0 the lowest. w_1 and w_2 are constrained by a constant value of 1 to reflect the complementary relationship between the two maps, that is, $w_1 + w_2 = 1$. On an overlaid map of P_1 and P_2, every site or area bears a pair of capability scores (c_{1i}, c_{2j}), and has a suitability score value S_{ij} given by

$$S_{ij} = c_{1i}w_1 + c_{2j}w_2. \qquad (1)$$

Or equivalently,

$$S_{ij} = (c_{1i} - c_{2j})w_1 + c_{2j}. \qquad (2)$$

Since $0 \leq w_1, w_2 \leq 1$ and $0 \leq c_{1i}, c_{2j} \leq 1$, we have $S_{ij} \in [0, 1]$. Equation (2) represents a family of 9 linear functions, namely the S_{ij} family of land suitability functions (see Table 1). Each member of the family corresponds to a unique pair of capability values (c_{1i}, c_{2j}), and falls into one of the two categories upon the presence of the independent variable w_1 (Table 1). Geometrically, these functions are either horizontal lines (Category I) or slant lines (Category II) as shown in Figure 1.

Table 1 Here.

Figure 1 Here.

The intersecting points of these lines have one of the following five w_1 values $\{0, 1/3, 1/2, 2/3, 1\}$. These five w_1 values divide the weight value range $[0, 1]$ into 4 disjoint open intervals (Table 2). These open intervals and the five dividing w_1 values $\{0, 1/3, 1/2, 2/3, 1\}$ are then called *segments* of $[0, 1]$. Each segment comprises a set of weight values that satisfy the following two conditions. First, for any two weight values from such a set, the orders of dominance among the (land suitability) functions produced by these two values are identical. Second, the orders of dominance among the (land suitability) functions produced by weight values from two different such sets are never identical. For example, segment $(0, 1/3)$ preserves an order of dominance $S_{11} > S_{21} > S_{31} > S_{12} > S_{22} > S_{32} > S_{13} > S_{23} > S_{33}$ among the 9 functions that does not appear in any other segments (Table 2). This unique ordinal set can be visualized by the order of the nine lines above segment $(0, 1/3)$ in Figure 1.

This phenomenon of weight segmentation has been cited, though implicitly, in justifying the usage of approximate weights (for example, Kirkwood and Corner, 1993, p.457) and differential weights (for example, Barron, 1988, p.142). In our case, it presents a simple solution to the problems of redundancy and under-representation: designing buttons with the way the weight value range is segmented. Following this principle of design with segmentation (Xiang and Salmon, 2001), a button system for the overlay of two parent-maps should have 9 tabs, each representing a weight segment on $[0, 1]$ in Table 2.

Table 2 Here.

3. WEIGHT SEGMENTATION IN THE THREE PARENT-MAP CASE

We now examine the phenomenon of weight segmentation in the presence of three parent-maps. Let c_1, c_2, and c_3 be the attributes of the parent-maps P_1, P_2, and P_3, respectively; $\{c_{3k}\}$ be an ordered set of land capability values of c_3 ($k = 1, 2, 3$) with $c_{31} = 1$ representing the most capable, $c_{32} = 0.5$ the moderately, and $c_{33} = 0$ the least; $\{c_{1i}\}$ and $\{c_{2j}\}$ be $\{c_{3k}\}$'s counterparts for c_1 and c_2, respectively, and bear the same definitions as those in the 2 parent-map case in Section 2; w_1, w_2, and w_3 be the attribute weights for c_1, c_2, and c_3, respectively, representing their importance to the assessment; w_1, w_2, and w_3 be all defined on the weight value range $[0, 1]$ where a value of 1 represents the highest level of importance and 0 the lowest; w_1, w_2, and w_3 be constrained by a constant value of 1 to reflect the complementary relationship among the three maps, that is, $w_1 + w_2 + w_3 = 1$.

On an overlaid map of P_1, P_2, and P_3, every site or area carries a trio of capability values $(c_{1i}, c_{2j}, c_{3k)})$, and has a suitability score S_{ijk} given by

$$S_{ijk} = c_{1i}w_1 + c_{2j}w_2 + c_{3k}w_3. \tag{3}$$

Equation (3) can be transformed into an equivalent form

$$S_{ijk} = (c_{1i} - c_{3k})w_1 + (c_{2j} - c_{3k})w_2 + c_{3k} \tag{4}$$

by replacing w_3 with $1 - w_1 - w_2$. Here, $w_1 \geq 0$, $w_2 \geq 0$, and $w_1 + w_2 \leq 1$. This transformation, by eliminating the dependent variable w_3, simplifies the computation process, and, as we shall demonstrate next, makes a visualization of the weight segmentation possible.

Equation (4) represents a family of 27 linear functions, namely the S_{ijk} family of suitability functions. Each function corresponds to a unique triple of capability values (c_{1i}, c_{2j}, c_{3k}), and belongs to one of the three categories classified by the number of independent variables in it (Table 3).

Table 3 Here.

Geometrically, a member of the S_{ijk} family is a plane defined over the triangular domain $\Delta = \{(w_1, w_2, 0) : \ w_1 \geq 0, w_2 \geq 0, w_1 + w_2 \leq 1\}$. The three S_{ijk} functions in Category I carry no variable (see Table 3) and therefore are planes parallel to the w_1w_2 plane. Figure

2(a) shows the function S_{222} over the domain Δ. The twelve S_{ijk} functions in Category II carry one variable, either w_1 or w_2. They are planes that contain either the w_1 or the w_2 axis. Figure 2(b) shows the function S_{133} over the domain Δ. The twelve S_{ijk} functions in Category III contain both w_1 and w_2. These are slant planes that only intersect the w_1 and w_2 axes at a single point. Figure 2(c) shows the function S_{113} over the domain Δ.

Figure 2 Here.

The intersection of any two such planes is a line. Because of their monotonic nature, one plane is over the other plane on one side of this line and below the other plane on the other side of this line. To illustrate, consider functions S_{222} and S_{113}. The former is parallel to the base Δ (Figure 2(a)), and the latter slanted away from the axis S (Figure 2(c)). Along the intersecting line (shown in Figure 3 as the dashed line on S_{222}), which is defined by $w_1+w_2 = 0.5$, $S = 0.5$, they share a same land suitability value (that is, $S_{222} = S_{113} = 0.5$). Off the line, however, function S_{222} dominates S_{113} (that is, $S_{222} > S_{113}$) on one side of the line where $w_1 + w_2 < 0.5$, but becomes dominated by S_{113} (that is, $S_{222} < S_{113}$) on the other side where $0.5 < w_1 + w_2 \leq 1$.

Therefore, the weight value pairs that define the intersections have a weight-segmenting effect on Δ. They divide Δ into segments. In general, when all 27 S_{ijk} functions are considered, a segment is a region, or a line, or a point on Δ that supports one and only one unique order of dominance among the S_{ijk} functions. By "support", it is meant that all the pairs of weight values within a segment, once incorporated into the 27 S_{ijk} functions, will produce an identical ordinal set of suitability values that is unique to the segment. For instance, the weight value pairs (w_1, w_2) that define the intersecting line in the above example, that is, the intersection of the line $w_1 + w_2 = 0.5$ and Δ, divide Δ into three segments (Figure 3) in the case when only these two functions are considered. Each supports a unique order of dominance between the two functions S_{222} and S_{113}.

Figure 3 Here.

When all 27 S_{ijk} functions are considered, there are 499 segments on Δ, representing a significant increase from 9 in the two parent-map case (see Section 2). These segments are identified through a mathematical procedure developed in this research. Conceptually, it first searches for all the intersection lines among the 27 functions, and then projects them onto Δ to locate their corresponding or defining weight value pairs (w_1, w_2), such as those in the above example that comply with $w_1 + w_2 = 0.5$. Thirdly, it uses these weight value pairs to delineate the segment boundaries on Δ. Figure 4 provides a graphical rendering of these segments while Table 4 lists all the weight-segmenting lines.

Table 3 Here.

Figure 4 Here.

4. THE TYPES OF WEIGHT SEGMENTS AND THEIR VALUE REPRESENTATIONS

As stated in the introduction of this article, our focus of investigation is on two pieces of knowledge that are essential to button design for the weighting of three parent-maps. That is, the total number of segments and their value representations. Since we know from the last section that there are a total of 499 segments, we shall concentrate on the value representations of these segments, that is, what value(s) should be used to represent each segment?

The 499 segments fall into 3 categories: (a) an open region bounded by the line intervals shown in Figure 4 such that its interior does not contain any point from these line intervals, (b) an open line interval whose end points intersect one or more other line intervals but not its interior points, or (c) an intersecting point of these line intervals. We call these segments type A, B and C segments respectively. Examples of these segments are shown in Figure 4 , where the region with two $*$ signs indicates a type A segment, the double line interval indicates a type B segment and the dot indicates a type C segment. Notice that in the case of two parent-maps (Section 2), there are only types B and C segments. A type A segment is of dimension 2, a type B segment is of dimension 1 and a type C segment is of dimension 0. Readers should also notice in the figure how the three "segments" in the example considered in Section 3, that is, the line defined by $w_1 + w_2 = 0.5$, the triangle by $0 \leq w_1 + w_2 < 0.5$, and the trapezoid by $0.5 < w_1 + w_2 \leq 1$ (see Figure 3), are subdivided into smaller segments by the projections of other intersection lines. A simple count shows that there are 156 type A segments, 249 type B segments and 94 type C segments.

In the following, we shall use several (w_1, w_2) pairs to show the differences and similarities of different types of segments. The positions of these points are shown in Figure 4 from Section 3. The corresponding orders of dominance among the S_{ijk} functions are listed in Table 5. The first two (w_1, w_2) pairs are $(0.35, 0.4)$ and $(0.34, 0.38)$, they are both marked with an $*$ in Figure 4. As one can tell, they are from the same segment since they are in the same open region bounded by the line intervals (which are the projections of the intersecting lines of the S_{ijk} functions). The first and third columns in Table 5 are the S_{ijk} function values (in the ascending order) and the second and fourth columns are their corresponding S_{ijk} functions. That is, columns 2 and 4 are the orders of dominance of the S_{ijk} functions with respect to these two pairs. We see that they are identical. On the other hand, $(w_1, w_2) = (0.55, 0.28)$ is a point in Δ that is in a different segment (also of type A). This point is marked with a # in Figure 4. The S_{ijk} function values at this point and the corresponding order of dominance are listed in columns 5 and 6 of Table 5. Observe that although this order of dominance is different from that of $(0.35, 0.4)$ and $(0.34, 0.38)$, it does not contain any equal function values, just as those corresponding to $(0.35, 0.4)$ and $(0.34, 0.38)$. Next, we consider the point $(w_1, w_2) = (0.56, 0.28)$. It is marked with a $\circ$ in Figure 4. Notice that this point is in a type B segment. That is, it is on one (but just one) line interval. The S_{ijk} function values (and the corresponding functions in the ascending order of dominance) are in columns 7 and 8 of Table 5. At first, one may expect that there would be only one pair of equal S_{ijk} function values here since there is only one line interval involved. But the values listed suggest otherwise. In fact, this line interval is the projections of several intersecting lines (only that these lines happen to have the same projection). Finally, we calculate the S_{ijk} function values at $(w_1, w_2) = (\frac{1}{2}, \frac{1}{3})$, which is a type C segment as shown in Figure 4 (marked with a solid dot). Its corresponding S_{ijk} function values and the order of dominance are listed in columns 9 and 10 of Table 5.

Table 5 Here.

5. CONCLUSIONS

From the above discussions, we are able to draw the following conclusions. First of all, to provide an adequate and non-redundant representation, a button system should have 499 tabs, of which 156 represent type A segments, 249 type B, and 94 type C. The existing button design for the weighting of 2 parent-maps (Xiang and Salmon, 2001) is thus most definitely inadequate. Secondly, to represent a type A (a region) or B (a line interval) segment, one can use any value within that segment; to represent a type C segment (a point), one has no choice but the value at the segment. Thirdly, one shall find himself entrapped in the same dilemma that he faces in the two parent-map case (Xiang and Salmon, 2001, p.659) should he follow the conventional odd-numbered and even-stepped approach to button design. On the one hand, if he uses a smaller number of buttons at a larger interval, say, the 66 evenly spaced buttons $(0, 0)$, $(0, 0.1)$, $(0.1, 0)$, $(0.1, 0.1)$, ..., (as shown in Figure 5), the problem of under representation becomes a serious issue. On the other hand, if he uses a large number of buttons, say, the 5151 tabs represented by $(0, 0)$, $(0, 0.01)$, $(0.01, 0)$, $(0.01, 0.01)$, ..., then he will inevitably worsen the redundancy problem. Even if the number of evenly spaced tabs used is very close to 499, these two problems still exist as shown in Figure 6, where 496 evenly spaced points are plotted in Δ. We suspect that the dilemma is inherent to the even-stepped approach. Although this is subject to further proof, we are quite confident that we will be able to provide such a proof in our future study.

Figure 5 here.

Figure 6 here.

The study also sheds some lights on the multiple parent-map case.

Firstly, it is theoretically possible to extend our method to the general case of multiple parent-maps. In the 2 parent-map case, we have dimension 0 (points) and dimension 1 (line interval) segments. In the three parent-map case, we have dimension 0, dimension 1 and dimension 2 (plane regions) segments. In general, if there are n parent-maps involved, then we will have n different types of segments, namely the ones from dimension 0, dimension 1, ... dimension $n-1$. For instance, if we have four parent-maps, then the weight range (after replacing w_4 by $1-w_1-w_2-w_3$) is the solid with the shape of a cut-off corner from a table. It has four sides as shown in Figure 7. This will be divided by planes that are the projections of the intersections of the super planes representing the land suitability functions (with w_1, w_2 and w_3 as variables). The results are then segments of solids (without boundary) which are of dimension 3, the faces of these solids (without boundary in their own) which are of dimension 2, the boundaries of these faces (line intervals) without end points which are of dimension 1, and the end points of these line intervals which are of dimension 0.

Secondly, the drastic increase in the number of segments from 9 in the 2 parent-map case to 499 in the 3 parent-map case hints that there is at least an exponential relationship between the number of parent-maps and that of segments on the overlaid map. This raises questions about not only the usability of a future button system that is designed with the segments, but also the practical legitimacy of the design with segmentation approach. How practical is it for people to use a button system with so many tabs? Does the design with segmentation approach impose a level of precision, represented by the number of tabs in a button system, that is valid theoretically but absent in people's minds (Barron and Barrett, 1996, p.1515; Hobbs, 1980, p.729; Xiang, 2001, p.62)? As the number of parent-maps climbs up, are we reaching the plateau of incompatibility where not only measurement precision and cognitive comprehension become mutually exclusive (Miller, 1956, p.86; Zadeh, 1973, p.28), but a pursuit for rigor is also likely unnecessary (Zadeh, 1973, p.29)? If so, should we switch from the precision-driven multi-valued logic, which underlies the design with segmentation approach, to the fuzzy logic that seeks a compromise between precision and relevance (Zadeh, 1973)? These questions require further investigation but at least suggest that our method not be extended to the multiple parent-map case without modifications.

Figure 5 Here.

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Category	(c_{1i}, c_{2j})	$S_{ij} = (c_{1i} - c_{2j})w_1 + c_{2j}$
I	$(1, 1)$	$S_{11} = 1$
	$(0.5, 0.5)$	$S_{22} = 0.5$
	$(0, 0)$	$S_{33} = 0$
II	$(1, 0.5)$	$S_{12} = 0.5(1 + w_1)$
	$(1, 0)$	$S_{13} = w_1$
	$(0.5, 1)$	$S_{21} = 1 - 0.5w_1$
	$(0.5, 0)$	$S_{23} = 0.5w_1$
	$(0, 1)$	$S_{31} = 1 - w_1$
	$(0, 0.5)$	$S_{32} = 0.5(1 - w_1)$

Table 1: The S_{ij} family of land suitability functions

Segment #	w_1 value range	Order of dominance (ordinal set of S_{ij} values)
1	0	$S_{11} = S_{21} = S_{31} > S_{12} = S_{22} = S_{32} > S_{13} = S_{23} = S_{33}$
2	$(0, 0.33)$	$S_{11} > S_{21} > S_{31} > S_{12} > S_{22} > S_{32} > S_{13} > S_{23} > S_{33}$
3	0.33	$S_{11} > S_{21} > S_{31} = S_{12} > S_{22} > S_{32} = S_{13} > S_{23} > S_{33}$
4	$(0.33, 0.5)$	$S_{11} > S_{21} > S_{12} > S_{31} > S_{22} > S_{13} > S_{32} > S_{23} > S_{33}$
5	0.5	$S_{11} > S_{12} = S_{21} > S_{13} = S_{22} = S_{31} > S_{23} = S_{32} > S_{33}$
6	$(0.5, 0.67)$	$S_{11} > S_{12} > S_{21} > S_{13} > S_{22} > S_{31} > S_{23} > S_{32} > S_{33}$
7	0.67	$S_{11} > S_{12} > S_{13} = S_{21} > S_{22} > S_{23} = S_{31} > S_{32} > S_{33}$
8	$(0.67, 1)$	$S_{11} > S_{12} > S_{13} > S_{21} > S_{22} > S_{23} > S_{31} > S_{32} > S_{33}$
9	1	$S_{11} = S_{12} = S_{13} > S_{21} = S_{22} = S_{23} > S_{31} = S_{32} = S_{33}$

Table 2: Segmentation of the weight value range $[0, 1]$ in the two parent-maps case

Category	(c_{1i}, c_{2j}, c_{3k})	$S_{ijk} = (c_{1i} - c_{3k})w_1 + (c_{2j} - c_{3k})w_2 + c_{3k}$
I	$(1, 1, 1)$	$S_{111} = 1$
	$(0.5, 0.5, 0.5)$	$S_{222} = 0.5$
	$(0, 0, 0)$	$S_{333} = 0$
II	$(1, 0.5, 0.5)$	$S_{122} = 0.5(w_1 + 1)$
	$(1, 0, 0)$	$S_{133} = w_1$
	$(0.5, 1, 1)$	$S_{211} = 1 - 0.5w_1$
	$(0.5, 0, 0)$	$S_{233} = 0.5w_1$
	$(0, 1, 1)$	$S_{311} = 1 - w_1$
	$(0, 0.5, 0.5)$	$S_{322} = 0.5(1 - w_1)$
	$(1, 0.5, 1)$	$S_{121} = 1 - 0.5w_2$
	$(1, 0, 1)$	$S_{131} = 1 - w_2$
	$(0.5, 1, 0.5)$	$S_{212} = 0.5(1 + w_2)$
	$(0.5, 0, 0.5)$	$S_{232} = 0.5(1 - w_2)$
	$(0, 1, 0)$	$S_{313} = w_2$
	$(0, 0.5, 0)$	$S_{323} = 0.5w_2$
III	$(1, 1, 0.5)$	$S_{112} = 0.5(1 + w_1 + w_2)$
	$(1, 1, 0)$	$S_{113} = w_1 + w_2$
	$(1, 0.5, 0)$	$S_{123} = w_1 + 0.5w_2$
	$(1, 0, 0.5)$	$S_{132} = 0.5(1 + w_1 - w_2)$
	$(0.5, 1, 0)$	$S_{213} = 0.5w_1 + w_2$
	$(0.5, 0.5, 1)$	$S_{221} = 1 - 0.5(w_1 + w_2)$
	$(0.5, 0.5, 0)$	$S_{223} = 0.5(w_1 + w_2)$
	$(0.5, 0, 1)$	$S_{231} = 1 - 0.5w_1 - w_2$
	$(0, 1, 0.5)$	$S_{312} = 0.5(w_2 - w_1 + 1)$
	$(0, 0.5, 1)$	$S_{321} = 1 - w_1 - 0.5w_2$
	$(0, 0, 1)$	$S_{331} = 1 - w_1 - w_2$
	$(0, 0, 0.5)$	$S_{332} = 0.5(1 - w_1 - w_2)$

Table 3: The S_{ijk} family of land suitability functions

#	Equation	#	Equation
1	$w_2 = \frac{1}{3} - w_1$	2	$w_2 = \frac{1}{2} - \frac{3}{2}w_1$
3	$w_2 = 1 - 3w_1$	4	$w_1 = \frac{1}{3}$
5	$w_2 = -1 + 3w_1$	6	$w_2 = \frac{1}{3} - \frac{2}{3}w_1$
7	$w_2 = \frac{1}{2} - w_1$	8	$w_2 = 1 - 2w_1$
9	$w_1 = \frac{1}{2}$	10	$w_2 = -1 + 2w_1$
11	$w_2 = \frac{1}{3} - \frac{1}{3}w_1$	12	$w_2 = \frac{1}{2} - \frac{1}{2}w_1$
13	$w_2 = \frac{1}{3}$	14	$w_2 = \frac{1}{2}$
15	$w_2 = \frac{1}{3} + \frac{1}{3}w_1$	16	$w_2 = \frac{1}{2} + \frac{1}{2}w_1$
17	$w_2 = \frac{1}{2}w_1$	18	$w_2 = w_1$
19	$w_2 = 2w_1$	20	$w_2 = \frac{2}{3} - \frac{3}{4}w_1$
21	$w_2 = 2 - 4w_1$	22	$w_2 = \frac{1}{2} - \frac{3}{4}w_1$
23	$w_2 = \frac{2}{3} - w_1$	24	$w_2 = 1 - \frac{3}{2}w_1$
25	$w_2 = 2 - 3w_1$	26	$w_1 = \frac{2}{3}$
27	$w_2 = \frac{2}{3} - \frac{2}{3}w_1$	28	$w_2 = \frac{1}{2} - \frac{1}{4}w_1$
29	$w_2 = \frac{2}{3} - \frac{1}{3}w_1$	30	$w_2 = \frac{2}{3}$

Table 4: The weight segmenting lines on Δ

$(w_1, w_2) =$ (.35, .4)		$(w_1, w_2) =$ (.34, .38)		$(w_1, w_2) =$ (.55, .28)		$(w_1, w_2) =$ (.56, .28)		$(w_1, w_2) =$ (.5, .33)	
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10
0	S_{333}	0	S_{333}	0	S_{333}	0	S_{333}	0	S_{333}
0.13	S_{332}	0.14	S_{332}	0.09	S_{332}	0.08	S_{332}	0.08	S_{332}
0.18	S_{233}	0.17	S_{233}	0.14	S_{323}	0.14	S_{323}	0.17	S_{323}
0.2	S_{323}	0.19	S_{323}	0.17	S_{331}	0.16	S_{331}	0.17	S_{331}
0.25	S_{331}	0.28	S_{331}	0.23	S_{322}	0.22	S_{322}	0.25	S_{322}
0.3	S_{232}	0.31	S_{232}	0.275	S_{233}	0.28	S_{313}	0.25	S_{233}
0.33	S_{322}	0.33	S_{322}	0.28	S_{313}	0.28	S_{233}	0.33	S_{321}
0.35	S_{133}	0.34	S_{133}	0.31	S_{321}	0.3	S_{321}	0.33	S_{313}
0.38	S_{223}	0.36	S_{223}	0.36	S_{232}	0.36	S_{312}	0.33	S_{232}
0.4	S_{313}	0.38	S_{313}	0.365	S_{312}	0.36	S_{232}	0.42	S_{223}
0.43	S_{231}	0.45	S_{231}	0.42	S_{223}	0.42	S_{223}	0.42	S_{312}
0.45	S_{321}	0.47	S_{321}	0.445	S_{231}	0.44	S_{311}	0.42	S_{231}
0.48	S_{132}	0.48	S_{132}	0.45	S_{311}	0.44	S_{231}	0.5	S_{311}
0.5	S_{222}	0.5	S_{222}	0.5	S_{222}	0.5	S_{222}	0.5	S_{222}
0.53	S_{312}	0.52	S_{312}	0.55	S_{133}	0.56	S_{213}	0.5	S_{133}
0.55	S_{123}	0.53	S_{123}	0.555	S_{213}	0.56	S_{133}	0.58	S_{221}
0.58	S_{213}	0.55	S_{213}	0.59	S_{221}	0.58	S_{221}	0.58	S_{213}
0.6	S_{131}	0.62	S_{131}	0.635	S_{132}	0.64	S_{212}	0.58	S_{132}
0.63	S_{221}	0.64	S_{221}	0.64	S_{212}	0.64	S_{132}	0.67	S_{123}
0.65	S_{311}	0.66	S_{311}	0.69	S_{123}	0.7	S_{123}	0.67	S_{212}
0.68	S_{122}	0.67	S_{122}	0.72	S_{131}	0.72	S_{211}	0.67	S_{131}
0.7	S_{212}	0.69	S_{212}	0.725	S_{211}	0.72	S_{131}	0.75	S_{211}
0.75	S_{113}	0.72	S_{113}	0.78	S_{122}	0.78	S_{122}	0.75	S_{122}
0.8	S_{121}	0.81	S_{121}	0.83	S_{113}	0.84	S_{113}	0.83	S_{121}
0.83	S_{211}	0.83	S_{211}	0.86	S_{121}	0.86	S_{121}	0.83	S_{113}
0.88	S_{112}	0.86	S_{112}	0.92	S_{112}	0.92	S_{112}	0.92	S_{112}
1	S_{111}	1	S_{111}	1	S_{111}	1	S_{111}	1	S_{111}

Table 5: The orders of dominance

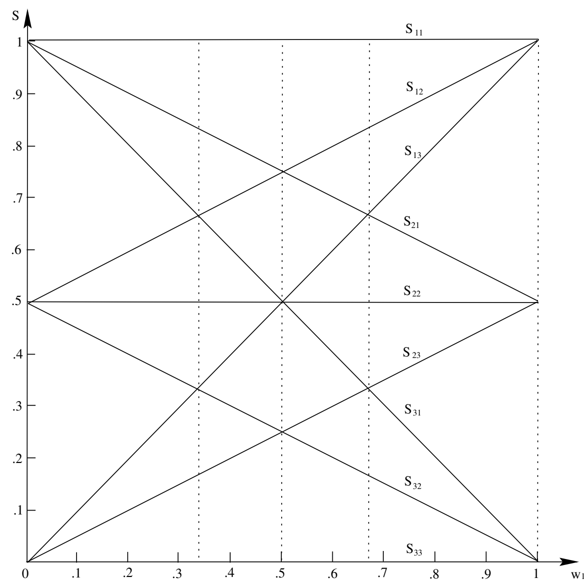

Figure 1: The relation between S_{ij} function values and the segments

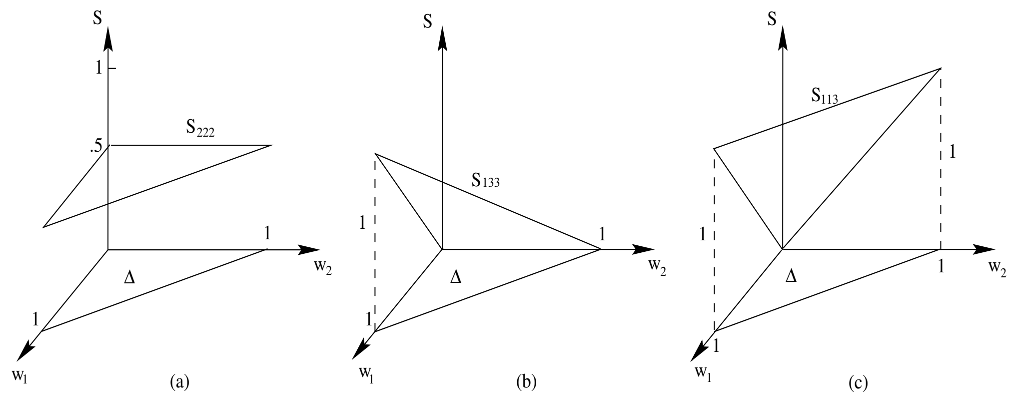

Figure 2: Illustrations of the S_{ijk} functions

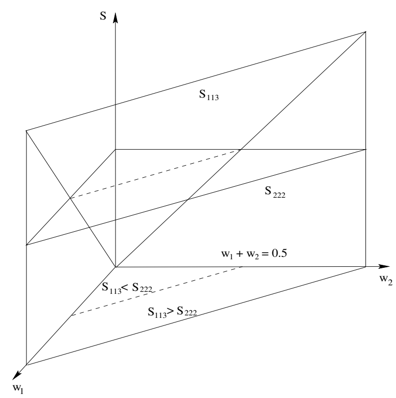

Figure 3: The intersection of S_{222}, S_{113} and the weight-segmenting effect of its corresponding weight value pairs

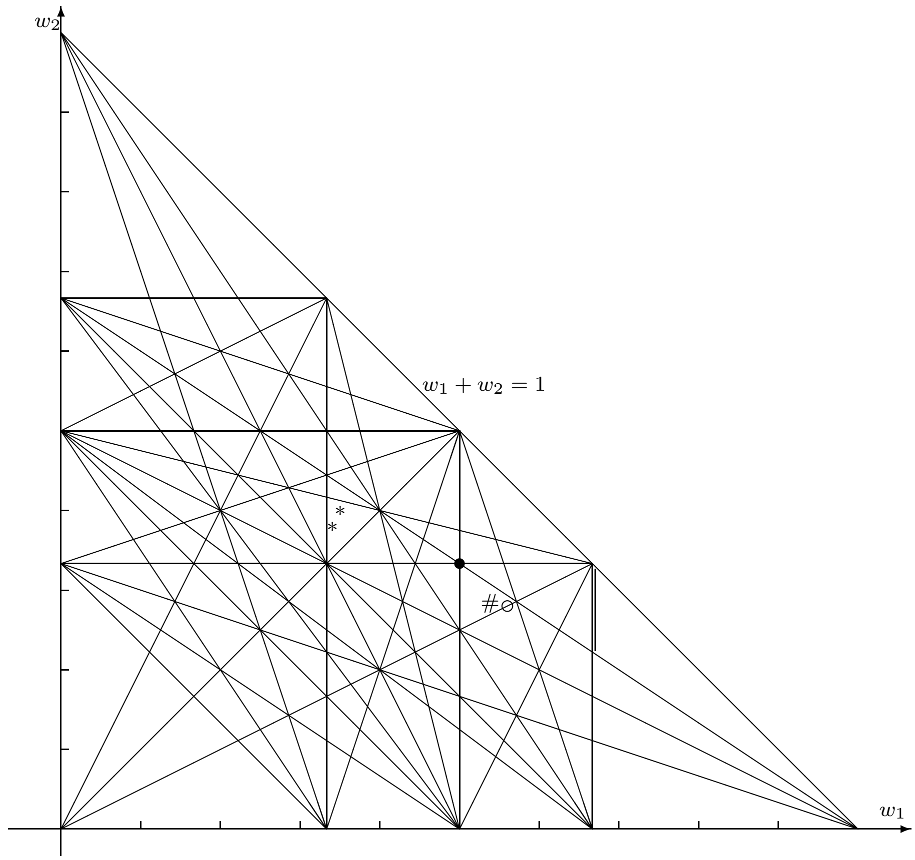

Figure 4: The segmentation of Δ

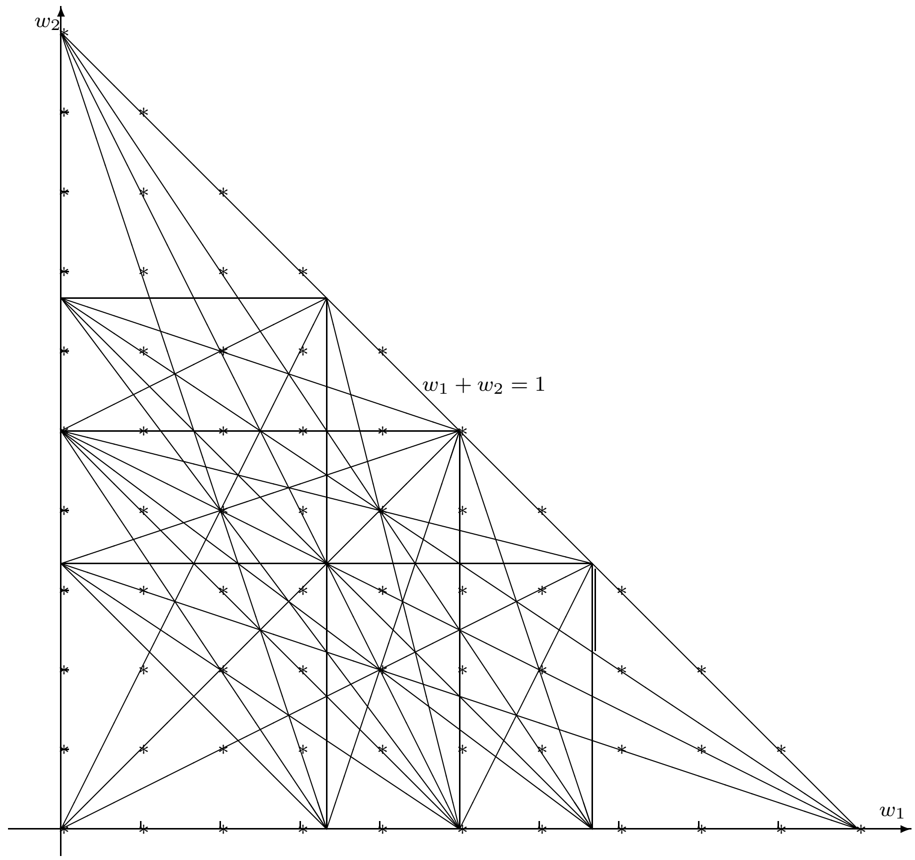

Figure 5: Evenly spaced points with interval length $\frac{1}{10}$

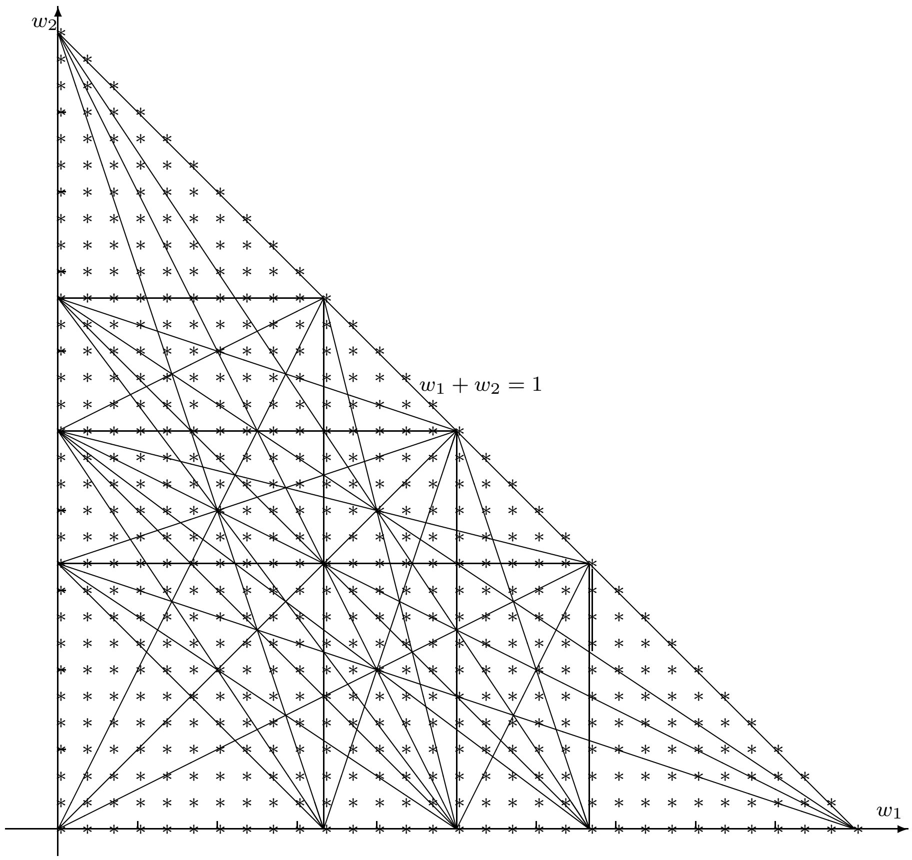

Figure 6: Evenly spaced points with interval length $\frac{3}{100}$

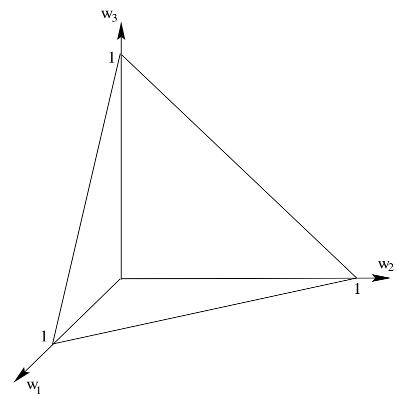

Figure 7: The weight range in the four parent-maps case