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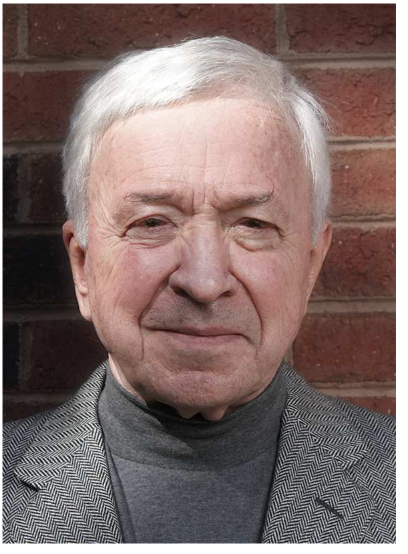
MATHEMATICAL LIFE

Stanislav Alekseevich Molchanov (on his 80th birthday)



Stanislav Molchanov was born on 21 December 1940 in the village of Snetinovo in the Ivanovo Oblast. His mother, Nina Grigorievna Molchanova, was an elementary school teacher. His father, Alexei Pavlovich Molchanov, was an accountant at a collective farm. As a son of a priest who had been persecuted after the Russian revolution, Alexei Molchanov was able to enroll at University (the Pedagogical Institute in the city of Ivanovo) only at the end of the 1940s, due to his status as a wounded veteran of WWII. After he graduated from the university, Alexei Molchanov started teaching physics at School no. 54 in the settlement of Nerl' in the Teikovo District of the Ivanovo Oblast. Nina Molchanova moved to work at the same school as a teacher of German.

After spending five years at a local school in Snetinovo, Molchanov moved to Nerl' with his parents, where he graduated from School no. 54 with a gold medal. While in high school, he studied on his own some parts of elementary mathematics and introductory calculus not included in the high school program, using brochures from the school mathematical library and a book by A. Ya. Khinchin.

In 1958 Molchanov enrolled in the Faculty of Mechanics and Mathematics at the Lomonosov Moscow State University. He received straight A grades, and, starting from his third year, he was a recipient of the Lenin Scholarship. He participated actively in the seminars of Prof. E. B. Dynkin, his scientific advisor. As a participant of the seminar, he carried out his first research work.

After graduating from the university, Molchanov stayed there as a graduate student from 1963 till 1966. He was a member of Dynkin's school, which exemplified a high level of general mathematical education: not only probability theory and stochastic processes, but also partial differential equations, Riemannian geometry, Lie groups and algebras, and so on.

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While in graduate school, Molchanov was involved in a high-profile educational initiative, the Mathematics School no. 2. Dynkin organized the first mathematics curriculum for this school, and Molchanov was his deputy for the programme and as editor of the journal *Matematicheskaya Shkola*, published not only for School no. 2, but also for A. N. Kolmogorov's Boarding School no. 18, and a number of other mathematics high schools in Moscow.

In 1967 Molchanov defended his Ph.D. thesis and (together with a group of fellow Ph.D. students) was offered a teaching position at Moscow State University, at a time when the Faculty of Mechanics and Mathematics underwent considerable expansion. In fact, he started working there already in 1966 (before defending his thesis) as an Assistant Professor at the Department of Probability Theory (chaired by Prof. B. V. Gnedenko). By that time Kolmogorov had already moved to the Interfaculty Laboratory, and a bit later became Head of the Department of Mathematical Logic.

In 1973 Molchanov was promoted to Associate Professor at the Department of Probability Theory. At this time his interest moved from pure probability theory to mathematical physics and spectral theory. This happened under the influence of V. I. Arnold on the one hand, and the physicists Academician Ya. B. Zeldovich and Academician I. M. Lifshitz on the other hand. In 1983 Molchanov defended his D.Sc. thesis on localization theory, but only in 1990 was he promoted to Full Professor. Over many years he had been responsible for a number of state order applied research projects.

In 1988 he started travelling abroad. His first trip was to attend the International Congress on Mathematical Physics in the UK, where he delivered a plenary lecture.

He was also an invited speaker at the International Congress of Mathematicians in Kyoto, Japan, in 1990.

In 1990 Molchanov received an invitation for one semester from the California Institute of Technology to work with Prof. B. Simon, one of the leading experts in mathematical physics. This is where he (together with Prof. R. Carmona) received a three-year NSF grant and was offered a job at the University of California at Irvine. After the collapse of the Soviet Union he stayed in the USA, while keeping his Russian citizenship and his scientific contacts with Russian colleagues. Since 1994 he has been Full Professor at the Department of Mathematics and Statistics at the University of North Carolina at Charlotte.

Molchanov is a Fellow of the American Mathematical Society. He has been invited to do research at leading European universities including Cambridge, Paris VI, Paris VII, La Sapienza (Rome), the University of Bonn, Bielefeld University, and so on. He is a member of the Editorial Boards of several mathematical journals. He is the Academic Advisor of the International Laboratory of Stochastic Analysis and its Applications at the Higher School of Economics in Moscow. He has been the Principal Investigator and Co-investigator in several NSF grants (in the USA) and Russian Science Foundation grants (in Russia). All of his work on NSF grants, as well as a significant share of his research papers published since the late 1990s, has been done in close collaboration with Prof. B. R. Vainberg from the same department at the University of North Carolina in Charlotte. Combining concepts from the theory of parabolic equations (Molchanov) and wave equations (Vainberg) has been particularly productive in applied work, such as optics, demography, biophysics, and so on.

More than 60 students have received their Ph.D. (or even D.Sc.) degrees with Molchanov as their advisor. His former students work at universities in Russia, the USA, Israel, and the UK. One of his students was Alexander Gordon [31], whose extraordinary mathematical talent flourished in large part thanks to Molchanov's help and guidance throughout Gordon's career.

A short review of Molchanov's research activity. The total number of his publications is difficult to establish, especially, if his papers in *Matematicheskaya Shkola* and classified reports (for instance, the ones containing the results of state order applied research projects) are to be taken into account. A conservative estimate is more than 300. We present a review of his most significant work. The topics of his research have developed in time from pure probability theory to mathematical physics and applied mathematics in a broad sense.

The early works, during the 1960s, pertained to the theory of diffusion processes and Martin boundaries (harmonic functions associated with Markov processes). This topic, suggested by Dynkin as his Ph.D. advisor, formed the basis of Molchanov's Ph.D. thesis (1967). In the paper [1], influenced by Arnold and recommended by him for publication in *Uspekhi Matematicheskikh Nauk*[1] (1975), we can already observe a transition from probability theory to classical spectral theory.

In the mid-1970s Molchanov started working on the spectral theory of the random Schrödinger operator, homogenization theory, intermittency effects, and other topics. Physical applications included the physics of disordered solid state (stationary random media) and the theory of physical processes in turbulent (non-stationary) media, mainly in magneto-hydrodynamics. On the first topic he worked with the group around Lifshitz (Academician L. A. Pastur and others), on the second topic with the astrophysics group around Zeldovich (A. A. Ruzmaikin and D. D. Sokolov). The number of publications in this area is also difficult to estimate because they often overlapped with adjacent areas of physics and mathematics, but it is definitely in excess of 100. Let us highlight the four publications [2]–[5] on one-dimensional localization theory from the early stages of studying the spectral properties of the random Schrödinger operator. These articles have a particulary large number of citations, especially [2] and [4], which lie at the foundation of random media theory.

Molchanov delivered a lecture course on random media theory at two high-profile summer schools on probability theory, the Taniguchi School (Japan, 1990) and the Saint-Flour School (France, 1992). He also delivered lecture courses in the USSR for scientists at several institutes of the USSR Academy of Sciences (Moscow, in the 1980s).

The USSR lectures were partly published in [10]. The Saint-Flour lectures were published in the Springer series [12], followed by the slightly revised version [27], also published by Springer.

Among the numerous papers by Molchanov on the multidimensional Anderson model (the lattice Laplacian plus a random potential), let us highlight the important

[1] Translated into English as *Russian Mathematical Surveys*.

paper [11] (with M. Aizenman), where the theorem about a pure point spectrum (localization) was proved in a very general setting for a large class of graphs and a non-local Laplacian, but under the same assumption of large disorder as previously. The so-called Aizenman–Molchanov method has its origin in that paper. It has been significantly developed and widely applied in the spectral theory of random operators.

Starting from the mid-1990s, Molchanov's research interests have gradually turned to deterministic models of spectral theory, in which ideas involving fractals were developed, martingale methods were used, connections with non-linear integrable equations were investigated, and so on; see [8], [15], [18], [17]–[20]. Note the paper [20] (with M. V. Novitskii and Vainberg), in which a new proof of the spectral L^2-hypothesis was obtained by using the KdV technique: the one-dimensional Schrödinger operator with square integrable potential has a pure absolutely integrable spectrum (up to rank one perturbations, for instance, for almost all boundary conditions at zero if the operator is defined on the half-axis). Some generalizations of this result connected with higher KdV integrals were also presented in this paper.

Molchanov's work in the theory of non-stationary random environment started with the problem of kinematic dynamo, which had to explain the generation of strong magnetic fields in the photosphere of hot stars. In the 1980s and 1990s this fundamental topic in astrophysics was at the centre of attention of several groups of physicists (Zeldovich in Moscow, K. Moffat in Cambridge, and others). During the 1990s Molchanov, together with Ruzmaikin and Sokolov, published about 10 articles on the kinematic dynamo and on the theory of temperature fluctuations on the ocean surface, which is technically linked to astrophysics. Some of this research, published in physics journals, did not reach the level of full mathematical rigour, particularly the parts relating to the infinite-dimensional theory of stochastic partial differential equations. All this work was summarized in the survey [7], which was reissued (with small additions) in Cambridge on the occasion of the 100th anniversary of Zeldovich's birthday. The problem of 'mathematizing' some of the results in [7] and [28] was partly solved in [13] (joint work with Carmona), but only for scalar fields.

The important new concept of *intermittency* has been introduced into mathematics after [7] and [28]. This concept is diametrically opposite to homogenization, according to which we can replace random or periodic media by homogeneous media in certain situations. An environment is intermittent if physical processes in it are determined by *rare but very strong fluctuations*. A typical example in astrophysics is the magnetic field of the Sun, whose energy is concentrated almost entirely in black spots, where the intensity of the field is hundreds of times the mean over the surface. Similar phenomena can be observed in demography.

The first mathematical results on intermittency were published by Molchanov with his German colleagues J. Gärtner and W. König, and also with Carmona and his graduate students. The most important works on intermittency itself are the report [13] mentioned above and the papers [9], [14], [16], and [25].

Solutions of the parabolic Anderson equation in stationary and non-stationary random media are homogeneous fields on the lattice $\mathbb{Z}^d$ (if time is fixed). It is natural to study sums of such (weakly dependent) random variables over a system of increasing cubes. This makes it possible to describe the transition from quenched to annealed probabilities, which involve averaging of the randomness of the medium. Similar questions arise in physical applications and often lead to bifurcations with respect to the parameters of the system. This topic has attracted the attention of experts, and there are many publications in the area. We note the original papers [21], [22], and [32].

In the USSR, USA, and contemporary Russia Molchanov has been a regular recipient of grants (occasionally more than one) for applied research awarded by various institutions such as the National Science Foundation in the USA or the Russian Science Foundation in Russia. Here we list some publications on applied topics. The survey [6] relates to work on the aging theory for polymer insulation materials (there are 7 publications in this area). The papers [24] and [23] are related to optics, a topic developed at the University of North Carolina in Charlotte (optic computers and the like; 10 publications). The articles [30], [33], and [34] pertain to population dynamics. More than 20 articles were published in total in this field, many of them in *Mathematical Population Studies* (*An International Journal of Mathematical Demography*), the best known journal in this area. Molchanov works in this field in conjunction with several colleagues: Vainberg in the USA, Y. Kondratiev and his group in Germany, O. Hryniv in Great Britain, E. O. Chernousova at the Moscow Institute of Physics and Technology in Russia, and E. B. Yarovaya and her group at Moscow State University in Russia.

The final important cycle of articles by Molchanov is related to theoretical biophysics, namely the problem of phase transition of protein molecules from the globular state to diffusion state (the standard example of such a transition is a boiled chicken egg). Molchanov has published 10 papers in this field. The most notable are [29] and [26].

Among other fields we can mention 'molecular motors', fractals, stochastic processes on the affine groups Aff($\mathbb{R}$), and random geometric progressions.

From the bottom of our hearts we wish Stanislav Molchanov good health and continuing success in his research and teaching activities.

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