Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. Find $\int_1^2 (2x+1)\,dx$.

(a) 0

(b) 2

(c) 3

(d) 4

(e) 8

2. Find $\int 2e^u\,du$.

(a) $2e^u + C$

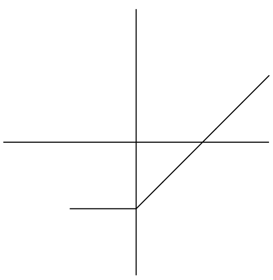
(b) $\dfrac{2}{u+1}e^{u+1} + C$

(c) $2\ln u + C$

(d) $2\cos u + C$

(e) $0 + C$

3. Consider the following graph of a function f:



Find $\int_{-1}^2 f(x)\,dx$.

(a) -1

(b) $-1/2$

(c) $-3/2$

(d) 0

(e) 1

4. Find $\int_0^1 (3x-1)^3\,dx$.

(a) 0

(b) 17/12

(c) 9

(d) 5/2

(e) 5/4

5. Find $\int_1^2 x^2 e^{x^3}\,dx$.

(a) $\frac{8}{3}e^8$

(b) 0

(c) e^{16}

(d) $e^8 - e$

(e) $\frac{1}{3}(e^8 - e)$

6. Find $\int xe^{3x}\,dx$

(a) $\frac{1}{2}x^2 \cdot \frac{1}{3}e^{3x} + C$

(b) $xe^{3x} - \frac{1}{3}e^{3x} + C$

(c) $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

(d) $xe^{3x} + \frac{1}{3}e^{3x} + C$

(e) $\frac{1}{3}xe^{3x} + \frac{1}{9}e^{3x} + C$

7. Find $\int 3\sin 2x\, dx$.

(a) $3\cos 2x + C$

(b) $6\cos 2x + C$

(c) $\frac{3}{2}\sin^2 2x + C$

(d) $\frac{3}{2}\cos 2x + C$

(e) $-\frac{3}{2}\cos 2x + C$

8. Find $\int \sin^3 t\cos t\, dt$.

(a) $-\frac{1}{4}\cos^3 t + C$

(b) $-\cos^4 t + 3\sin t\cos^3 t + C$

(c) $\cos^4 t - 3\sin^2 t\cos^2 t + C$

(d) $\frac{1}{4}\sin^4 t + C$

(e) $-\frac{1}{4}\sin^4 t + C$

9. Use the Fundamental Theorem of Calculus to find $g'(1)$ if $g(x) = \int_0^x \ln(t^4 + 2)\, dt$.

(a) $-\frac{4}{3}$

(b) $\frac{4}{3}$

(c) $-\frac{1}{3}$

(d) $\frac{1}{3}$

(e) $\ln 3$

10. The power series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ is the Maclaurin expansion of which of the following?

(a) $\sin x$

(b) $\cos x$

(c) e^x

(d) $\sin x^2$

(e) $\cos x^2$

11. Find $\int x \cos x \, dx$.

(a) $x \sin x + \cos x + C$

(b) $x \sin x - \cos x + C$

(c) $x \cos x + \sin x + C$

(d) $x \cos x - \sin x + C$

(e) $\frac{1}{2}x^2 \sin x + C$

12. The following integral form appears in the Table of Integrals in your text:

$$\int \sqrt{a^2 + u^2}\, du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln(u + \sqrt{a^2 + u^2}) + C$$

Use this form and an appropriate substitution to find $\int \sqrt{1 + 9x^2}\, dx$

(a) $\dfrac{3x}{2}\sqrt{1 + 9x^2} + \dfrac{1}{2}\ln(3x + \sqrt{1 + 9x^2}) + C$

(b) $\dfrac{x}{2}\sqrt{1 + 9x^2} + \dfrac{1}{2}\ln(x + \sqrt{1 + 9x^2}) + C$

(c) $\dfrac{3x}{2}\sqrt{1 + 9x^2} + \ln(3x + \sqrt{1 + 9x^2}) + C$

(d) $\dfrac{x}{2}\sqrt{1 + 9x^2} + \dfrac{1}{6}\ln(3x + \sqrt{1 + 9x^2}) + C$

(e) $\dfrac{x}{6}\sqrt{1 + 9x^2} + \dfrac{1}{6}\ln(x + \sqrt{1 + 9x^2}) + C$

13. Find $\int_0^2 |x - 1|\, dx$.

(a) 0

(b) 1/2

(c) 1

(d) 3/2

(e) 2

14. Consider the sequence $\{a_n\}$, where $a_n = \frac{2n+1}{5n-1}$. Which of the following statements is correct?

(a) The sequence diverges to ∞.

(b) The sequence diverges, but not to ∞.

(c) The sequence converges to 1.

(d) The sequence converges to -1.

(e) The sequence converges to $\frac{2}{5}$.

15. The geometric series $0.27 + 0.0027 + 0.000027 + \cdots$

(a) converges to 27/100

(b) converges to 27/99

(c) converges to 1/3

(d) converges to 27

(e) diverges

16. Which of the following series converge?

$$\text{I. } \sum_{1}^{\infty} \frac{1}{n+1} \qquad \text{II. } \sum_{1}^{\infty} \sin n \qquad \text{III. } \sum_{1}^{\infty} \frac{1}{n^3+1}$$

(a) I only

(b) II only

(c) III only

(d) I and II

(e) II and III

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. The speed of a runner (in ft/s) increased steadily during the first two seconds of a race. The runner's speed at half-second intervals is given in the table. Based on the information given, which of the following is the best upper estimate for the distance traveled during these two seconds.

time(s)	0	0.5	1.0	1.5	2.0
velocity(ft/s)	0	6	10	13	17

(a) $\frac{29}{2}$ ft

(b) 29 ft

(c) 23 ft

(d) 46 ft

(e) 30 ft

2. Which of the following definite integrals gives the length of the curve $y = \tan x$, $0 \leq x \leq \pi/4$?

(a) $\int_0^{\pi/4} \sqrt{1 - \sec^4 x}\, dx$

(b) $\int_0^{\pi/4} \sqrt{1 + \sec^4 x}\, dx$

(c) $\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$

(d) $\int_0^{\pi/4} \sqrt{1 - \sec^2 x}\, dx$

(e) $\int_0^{\pi/4} \sqrt{1 + \sec^2 x}\, dx$

3. The improper integral $\displaystyle\int_2^{\infty} \frac{dx}{x-1}$

(a) converges to 1.

(b) converges to 0.

(c) converges to 2.

(d) diverges to ∞.

(e) diverges to $-\infty$.

4. Let $f(x) = 1/x$. Find the Riemann sum for f on the interval $[1, 3]$, using 4 subintervals of equal width and taking the sample points to be the right endpoints. (Round your answer to two decimal places.)

(a) 1.28

(b) 2.57

(c) 1.45

(d) 0.95

(e) 1.90

5. A 10-pound bag of sand begins leaking at a constant rate of 0.5 lb/s as soon as it is picked up. How much work is done by a person who raises the bag from the floor to a height of 6 ft if he raises it at a constant rate of 1 ft/s?

(a) 10 ft-lbs

(b) 60 ft-lbs

(c) 51 ft-lbs

(d) 57 ft-lbs

(e) 7 ft-lbs

6. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2^n}$. Which of the following results in a successful determination of the convergence or divergence of this series?

(a) The ratio test

(b) The divergence test

(c) Comparison with the terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

(d) Comparison with the terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$

(e) The alternating series test

7. Find the area of the (finite) region between the graphs of $y = x^2$ and $y = \sin \frac{\pi}{2}x$. (Round your answer to two decimal places.)

(a) 0.67

(b) 1.00

(c) 0.00

(d) 0.33

(e) 0.30

8. What is the interval of convergence for the following power series: $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$?

(a) $(-\infty, \infty)$

(b) $(0, 2)$

(c) $[0, 2)$

(d) $(0, 2]$

(e) $[0, 2]$

9. The region below the curve $y = 1/x^2$ and above the x-axis, $1 \leq x \leq 3$ is rotated about the x-axis. Find the volume of the resulting solid. (Round your answer to two decimal places.)

(a) 2.00

(b) 0.67

(c) 1.10

(d) 1.01

(e) 0.32

10. What is the approximation obtained when the trapezoid rule with $n = 2$ is used to approximate $y = \int_1^2 x^2\,dx$? (Round the answer to one decimal place.)

(a) 2.3

(b) 4.8

(c) 2.4

(d) 3.6

(e) 1.8

11. A particle is moving along the x-axis. Its velocity at time t is $3t^2$ m/s. How far does the particle travel for $0 \leq t \leq 2$?

(a) 24 m

(b) 12 m

(c) 8 m

(d) 6 m

(e) 2 m

12. For a certain function f, it is known that $f'(x) = 5x^4 - 2$ and $f(1) = 4$. Find $f(-1)$.

(a) 0

(b) 1

(c) 2

(d) 3

(e) 6

13. Let $f(x) = x^6$. What is the coefficient of $(x-1)^3$ in the Taylor expansion of f about $a = 1$?

(a) 60

(b) 20

(c) 6

(d) 1

(e) 1/6

14. In the partial fraction expansion $\dfrac{7x-2}{x^2-x} = \dfrac{A}{x} + \dfrac{B}{x-1}$, find A.

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. Use integration by parts to derive the reduction formula

$$\int (\ln x)^n \, dx = x(\ln x)^n - \int n(\ln x)^{n-1} \, dx.$$

2. The region bounded by the curves $y = 4x^2$, $x = 1$, and $y = 0$ is rotated about the line $x = 1$ to form a solid.

(a) Use the "disk" method to set up a definite integral which will give the volume of the solid. You do not need to evaluate the integral.

(b) Now use the method of "cylindrical shells" to set up another definite integral which will also give the volume of the solid. You do not need to evaluate the integral.

3. A tank is formed by rotating about the y-axis the region bounded by the curves $y = x^2$, $x = 0$, and $y = 1$. All distances are measured in meters. The tank sits on the ground and is full of water. Find the work done in pumping all the water out at the top of the tank. (The density of water is 1000 kilograms per cubic meter, and the gravitational constant is 9.8.)

4. Recall the power series expansion $\dfrac{1}{1+t} = \sum_{n=0}^{\infty}(-1)^n t^n = 1 - t + t^2 - t^3 + \cdots$, valid on the interval $(-1, 1)$.

(a) (6 points) Use the given expansion to find a power series expansion for $\ln(1+t)$.

(b) (4 points) Use the expansion you derived in part (a) to approximate $\ln(1.1)$ to within 10^{-4}.

5. The error estimate when Simpson's rule is used to approximate $\int_a^b f(x)\,dx$ is given by

$$|E_n| \leq K\frac{(b-a)^5}{180n^4},$$

where n is the (even) number of subintervals and K is an upper bound for $|f^{(4)}(x)|$ on $[a,b]$. (Recall that $f^{(4)}$ is the fourth derivative of f.)

If Simpson's rule with n subintervals is used to approximate $\displaystyle\int_1^2 \frac{dx}{x}$ (you do *not* need to do this!), how large must n be chosen to guarantee accuracy to within 10^{-8}? Recall that n must be even.

Key:

Part I.

1. d
2. a
3. a
4. e
5. e
6. c
7. e
8. d
9. e
10. b
11. a
12. d
13. c
14. e
15. b
16. c

Part II.

1. c
2. b
3. d
4. d
5. c
6. a
7. e
8. a
9. d
10. c
11. c
12. e
13. b
14. b