MATH 1242 – CALCULUS II   SPRING 2016

COMMON FINAL EXAMINATION

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: _____   First Name:_____
(Please PRINT)   (Please PRINT)

Student ID #:_____   Instructor: _____   Section:_____

This exam is divided into three parts. Calculators are not allowed on Part I. You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the test after the first hour, but you cannot use your calculator during this time. You may use your calculator ONLY after your exam proctor has announced that calculators are allowed on Parts II and III. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)

PART I
- Part I consists of 15 multiple choice problems. These problems must be answered without the use of a calculator.
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the Opscan sheets.
- Make sure that your name appears on the Opscan sheets and that you fill in the circles corresponding to your name in the format Last, First.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. $\int 7x^3 \, dx = ?$

(a) $\frac{21}{4}x^4 + C$

(b) $\frac{7}{3}x^3 + C$

(c) $7x^4 + C$

(d) $\frac{7}{4}x^4 + C$

(e) $21x^2 + C$

2. $\int_0^2 x\sqrt{x^2 + 1} \, dx = ?$

(a) $\frac{1}{3}(5^{3/2} - 1)$

(b) $\frac{2}{3}(5^{3/2} - 1)$

(c) $\frac{1}{6}(5^{3/2} - 1)$

(d) $5^{3/2} - 1$

(e) $2(5^{3/2} - 1)$

3. $\int 5\cos(3t) \, dt = ?$

(a) $5\cos(3t) + C$

(b) $15\cos(3t) + C$

(c) $\frac{5}{3}\cos(3t) + C$

(d) $15\sin(3t) + C$

(e) $\frac{5}{3}\sin(3t) + C$

4. $\int_0^1 x^2 e^{x^3+1}\, dx = ?$

(a) $e^2 - e$

(b) $\frac{1}{3}(e^2 - e)$

(c) e^2

(d) $3e^2$

(e) $3(e^2 - e)$

5. $\int_0^1 x\sin(x) = ?$

(a) $\sin(1)$

(b) $\sin(1) - \cos(1)$

(c) $-\dfrac{1}{2}\sin(1)$

(d) $\cos(1) + \sin(1)$

(e) $\cos(1) - \sin(1)$

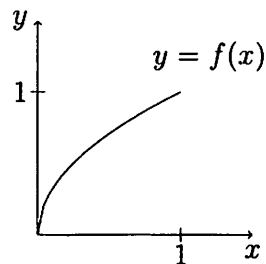6. $\displaystyle\int_0^1 \frac{4}{u+1}\, du = ?$

(a) 0

(b) -2

(c) $-\dfrac{1}{4}\ln(2)$

(d) 4

(e) $4\ln(2)$

7. Consider the graph of the function f :

Which of the following statements is correct?

(a) $\int_0^1 f(x)\,dx > 1$

(b) $\int_0^1 f(x)\,dx = 1$

(c) $0 < \int_0^1 f(x)\,dx < \frac{1}{2}$

(d) $\frac{1}{2} < \int_0^1 f(x)\,dx < 1$

(e) $\int_0^1 f(x)\,dx = 0$

8. If f is a function satisfying $f'(x) = 6e^{3x}$ and $f(0) = 5$, then $f(2) = $?

(a) $2e^6 + 3$

(b) $e^6 + 3$

(c) $2e^6$

(d) e^6

(e) $\frac{2}{3}e^6$

9. Find the area in the first quadrant bounded by the y-axis, the line $y = 1$, and the curve $y = x^2$.

(a) 1

(b) 0

(c) 1/2

(d) 1/3

(e) 2/3

10. $\int_9^\infty \frac{dt}{t^{3/2}} = ?$

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $-\frac{2}{3}$

(e) ∞

11. The series $0.4 + 0.04 + 0.004 + \cdots$

(a) diverges to ∞.

(b) diverges, but not to ∞.

(c) converges to 4/9.

(d) converges to 2/5.

(e) converges to 10/9.

12. The series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

(a) converges to a number between 0 and 1/4.

(b) converges to a number between 1/4 and 1/2.

(c) converges to a number between 1/2 and 3/4.

(d) converges to a number between 3/4 and 1.

(e) diverges.

13. If $f(u) = \int_1^u \sin(t^2 + 1)\, dt$, then $f'(2) = ?$

(a) $\sin(5)$

(b) $\cos(5)$

(c) $4\cos(5)$

(d) $4\sin(5)$

(e) $2\cos(5)$

14. The power series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ is the Maclaurin expansion of which of the following?

(a) e^{-x}

(b) $\sin(x)$

(c) $\cos(x)$

(d) $\tan^{-1}(x)$

(e) $\sec(x)$

15. The power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$

(a) has interval of convergence $(-\infty, \infty)$.

(b) has interval of convergence $[-2, 2]$.

(c) has interval of convergence $[-1, 1]$.

(d) has interval of converge $[-1/2, 1/2]$.

(e) converges only at $x = 0$.

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PART II

- Part II consists of 12 multiple choice problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
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Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. A particle is moving along the x-axis. If its velocity at time t is $\sqrt{t}$ m/s, how far does it travel for $0 \leq t \leq 4$?

(a) 32/3 m

(b) 4/3 m

(c) 2 m

(d) 8/3 m

(e) 16/3 m

2. Using the data of problem 1, find the average velocity of the particle for $0 \leq t \leq 4$.

(a) 16/3 m/s

(b) 1/2 m/s

(c) 4/3 m/s

(d) 2/3 m/s

(e) 8/3 m/s

3. Find the length of the curve $y = 1 - x^4$ for $0 \leq x \leq 1$. (Use your calculator to evaluate the definite integral involved. Round answer to one decimal place.)

(a) 0.8

(b) 1.0

(c) 1.2

(d) 1.4

(e) 1.6

4. Find the coefficient of $(x - 1)^3$ in the Taylor expansion of $f(x) = x^7$ about $a = 1$.

(a) 0

(b) 1/6

(c) 1/3

(d) 35

(e) 210

5. Find the volume of the solid formed when the region bounded above by the curve $y = e^x$ and below by the x-axis, $0 \le x \le 1$, is rotated about the x-axis. Round answer to two decimal places.

(a) 10.04

(b) 5.02

(c) 20.07

(d) 1.72

(e) 5.40

6. Consider the following table of values for a certain function f:

x	0	1/2	1	3/2	2
$f(x)$	1	-2	2	4	-4

(For example, $f(1) = 2$.) Find the Rieman sum for f on $[0, 2]$, using $n = 4$ subintervals of equal width and taking the sample points to be the left endpoints of the subintervals.

(a) 10

(b) 5

(c) 5/2

(d) 1/2

(e) 0

7. A 120-pound woman with a bag of sand on her back climbs up a rope as part of a military exercise. The bag weighs 50 pounds when she begins climbing. However, the bag has a small hole in it, which causes it to leak at a constant rate of 2 lb/ft as she climbs. How much work does she do if she climbs 25 feet?

(a) 625 ft-lb

(b) 312.5 ft-lb

(c) 3625 ft-lb

(d) 3937.5 ft-lb

(e) 4250 ft-lb

8. Find the area of the finite region in the first quadrant bounded by the curves $y = x^4$ and $y = 2 - x$.

(a) 1.4

(b) 1.3

(c) 1.2

(d) 1.1

(e) 1.0

9. Which of the following series converge(s)?

$$\text{I. } \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \qquad \text{II. } \sum_{n=0}^{\infty} \frac{3n+5}{n+7} \qquad \text{III. } \sum_{n=0}^{\infty} \frac{n+100}{n^3+1}$$

(a) I and III only

(b) I and II only

(c) II and III only

(d) I, II, and III

(e) I only

10. Find B in the partial fraction decomposition $\dfrac{3x+1}{x(x-1)} = \dfrac{A}{x} + \dfrac{B}{x-1}$.

(a) $B = -1$

(b) $B = 0$

(c) $B = 3$

(d) $B = 3/2$

(e) $B = 4$

11. Consider the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{\ln(n)}{n}$. Which of the following results in a successful determination of the convergence or divergence of this series?

(a) the ratio test

(b) the divergence test

(c) comparison with the terms of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^2}$

(d) the integral test

(e) the alternating series test

12. Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \dfrac{5^n x^n}{n!}$.

(a) $(-\infty, \infty)$

(b) $(-1/5, 1/5)$

(c) $(-1/5, 1/5]$

(d) $[-1/5, 1/5)$

(e) $[-1/5, 1/5]$

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For Grading Use Only:

Problem	1	2	3	4	5
Grade					
Out of					

Free Response Score [   ]

PART III
- Part III consists of 5 free response problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- Please show all of your work on the problem sheet provided. Work that is done on scratch paper or any other sheets will not be graded.
- You may use your calculator to check your answers, but complete justification must be shown for each problem. This includes all graphs, calculations and references to supporting theorems.
- Make sure that your name appears on each page of the test booklet.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. Find the x-coordinate of the centroid of the region in the first quadrant bounded above by the curve $y = 1 - x^2$ and below by the x-axis. Use you calculator to evaluate any definite integrals involved.

2. Find the volume of the solid formed when the region in the first quadrant bounded above by the curve $y = \sin(x)$ and below by the x-axis, $0 \leq x \leq \pi$, is revolved about the y-axis. Use your calculator to evaluate any definite integrals involved.

3. A hemispherical tank of base radius 1 m sits with its base on the ground. The tank is full of water. Set up a definite integral that gives the work done in pumping all of the water out of the tank at the top of the tank. (The density of water is 1000 kg/m^3, and the gravitational constant is 9.8.) Evaluate the definite integral by hand.

4. (a) Use the expansion $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ (valid for all real numbers x) to obtain an expansion for e^{-x^2}.

(b) Approximate $e^{-0.5^2}$ using the first four terms of your expansion in part (a).

(c) Use the Alternating Series Estimation Theorem to give an upper bound on the error in your estimation in part (b).

5. (a) Use the Midpoint Rule with $n = 4$ subintervals to approximate $\int_0^1 \dfrac{1}{x+1} \, dx$. (Note: you must show your work in both parts of this problem.)

(b) If $|f''(x)| \leq K$ for $a \leq x \leq b$, and E is the error when the midpoint rule with n subintervals is used to approximate $\int_a^b f(x) \, dx$, then

$$|E| \leq \frac{K(b-a)^3}{12n^2}.$$

Use this to give an upper bound on the error in your approximation in part (a).