Algebra Prelim Syllabus

I. Modern Algebra
   A. Sets, functions, equivalence relations
   B. Groups
      1. Definition, examples, elementary properties
      2. Subgroups, normal subgroups
      3. Cyclic groups, classification of subgroups
      4. Direct product (or sum) of groups
      5. Permutation groups
      6. Isomorphism
      7. Cosets, Lagrange's theorem
      8. Homomorphisms, factor groups
      9. Abelian groups, Fundamental theorem of finite abelian groups
      10. Sylow theorems, groups of small order
   C. Rings
      1. Definition, examples, elementary properties
      2. Subrings, ideals, maximal and prime ideals
      3. Factor rings and homomorphisms
      4. Polynomial rings
      5. Integral domains and divisibility
      6. Fields, construction of fields

II. Linear Algebra
   A. Vector spaces, elementary properties and examples
   B. Subspaces
   C. Spanning sets, linear independence, bases, dimension
   D. Linear transformations, 1-1, onto, null and range spaces, rank and nullity
   E. Matrix representation, change of coordinates
   F. Determinants
   G. Eigenvalues, eigenvectors, diagonalizability
   H. Invariant subspaces, Cayley-Hamilton theorem
   I. Inner product spaces (real and complex)
   J. Orthogonality, projection, Gram-Schmidt process
   K. Linear operators on inner product spaces, adjoints
   L. Normal and self-adjoint operators
   M. Orthogonal diagonalizability

III. Advice: Study the proofs of the theorems!