

Real Analysis II: MATH 8144

August 2023 Exam

Text: Real Analysis, 4th edition, Prentice Hall, H.L. Royden and P.M. Fitzpatrick

Topology and metric spaces

- Basic definitions and structures (11.1, 9.1, 9.2, 11.2)
- Sequences, separability, completeness (11.3, 9.3, 9.4, 9.6)
- Continuity (11.4, 9.3)
- Compactness (11.5, 9.5)
- Arzelà-Ascoli (10.1)
- Baire Category Theorem (10.2)
- Contraction Mapping Principle (10.3)

Normed linear spaces

- Definitions, including Banach space (13.1)
- Examples, including L^p spaces (7.1-7.4)
- Linear operators (13.2)
- Compactness and infinite dimensions (13.3)
- Open Mapping and Closed Graph theorems (13.4)
- Uniform Boundedness Principle (13.5)

Linear functionals

- Definitions (14.1)
- Riesz Representation Theorem for L^p spaces (8.1)
- Hahn-Banach Theorem (14.2)
- Weak topologies (14.3)

Hilbert spaces and continuous linear operators

- Definitions and first properties (16.1)
- Dual space (16.2)
- Bessel's inequality and orthonormal bases (16.3)
- Adjoints and symmetry for linear operators (16.4)
- Compact operators (16.5)
- Hilbert-Schmidt Theorem (16.6)