

Syllabus for the Ph.D. Qualifying Exam (STAT 5126/5127)

Textbook: Statistical Inference, 2nd ed., by George Casella and Roger L. Berger; Chapters 1-9

The level of problems for the Qualifying Exam is likely to be comparable to the exercises and examples in the textbook.

Topics covered by STAT 5126

- Probability theory
 - Probability, counting
 - Conditional probability, independence
 - Random variables, distribution functions, density and mass functions
- Transformations and expectations
 - Distributions of functions of a random variable
 - Expected values, moments, moment generating functions
- Common families of distributions
 - Discrete and continuous distributions
 - Exponential families, location and scale families
- Multiple random variables
 - Joint, marginal and conditional distributions
 - Independence, covariance, correlation
 - Multivariate distributions, transformations
 - Hierarchical models
- Properties of a random sample
 - Sum of random variables
 - Sample from Normal distribution, t and F distributions
 - Order statistics
 - Convergence of random variables, LLN, CLT, Delta method

Topics covered by STAT 5127

- Principles of data reduction
 - Sufficient statistics, Factorization theorem
 - Minimal sufficient statistics, ancillary statistics
 - Complete sufficient statistics, Basu's theorem
 - Exponential family
- Point estimation
 - Methods of finding estimators: Moments, MLE, Bayes

- Uniform minimum variance unbiased estimator (UMVUE)
- Cramer-Rao lower bound (CRLB)
- Rao-Blackwell theorem, Lehmann-Scheffe theorem
- Loss function optimality

- Hypothesis testing
 - Methods of finding tests: Likelihood ratio test (LRT),
 - Two types of error, power function, size, p-value
 - Most powerful test, Neyman-Pearson lemma
 - Monotone likelihood ratio (MLR), uniformly most powerful (UMP) test

- Interval estimation
 - Methods of finding interval estimation: Inverting a test statistic, pivotal quantities, pivoting the CDF, Bayesian intervals.
 - Methods of evaluating interval estimators: Size and coverage probability, shortest pivotal interval