

PART I (Calculators Not Allowed)

1. Evaluate the integral $\int_0^1 (8x - 3) dx$.

- (a) $\frac{7}{9}$ (b) $\frac{7}{3}$ (c) 1 (d) 2 (e) 3

2. Evaluate the integral $\int_0^2 |3x - 3| dx$.

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

3. Evaluate the integral $\int \frac{1 + \cos^2 x}{\cos^2 x} dx$.

- (a) $1 + \tan x + C$ (b) $x + \tan x + C$ (c) $x \tan x + C$ (d) $x \sec x + C$ (e) $x + \sec x + C$

4. Evaluate the integral $\int_1^2 4x \ln x dx$.

- (a) $8 \ln 2 - 3$ (b) $8 \ln 2 - 2$ (c) $8 \ln 2 - 1$ (d) $4 \ln 2 - 1$ (e) $4 \ln 2 - 2$

5. Evaluate the integral $\int_{-1}^2 4xe^{x^2} dx$.

- (a) $2e^2 - 2e$ (b) $2e^2 + 2e^{-1}$ (c) $2e^2 - 2e^{-1}$ (d) $2e^4 - 2e$ (e) $2e^4 - 2e^{-1}$

6. Evaluate the definite integral $\int_0^{\pi/2} x \sin(3x) dx$

- (a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) $-\frac{\pi}{9}$ (d) $\frac{\pi}{9}$ (e) $-\frac{1}{6}$

7. Evaluate the integral $\int e^{-2x+2} dx$.

- (a) $2e^{-2x} - 2x + C$
(b) $2e^{-2x} + C$
(c) $\frac{e^{-2x} + 2}{2} + C$
(d) $-\frac{e^{-2x+2}}{2} + C$
(e) $-2e^{-2x+2} + C$

8. Evaluate the integral $\int \frac{\sin\left(\frac{\pi}{x}\right)}{x^2} dx$.

- (a) $\frac{\sin\left(\frac{\pi}{x}\right)}{x} + C$ (b) $\frac{\cos\left(\frac{\pi}{x}\right)}{x} + C$ (c) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi x^2} + C$ (d) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi x} + C$ (e) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi} + C$

9. Evaluate the integral $\int \frac{3}{(x+7)^{3/2}} dx$

- (a) $\frac{6}{\sqrt{x+7}} + C$
(b) $-\frac{6}{\sqrt{x+7}} + C$
(c) $\frac{3}{\sqrt{x+7}} + C$
(d) $-\frac{3}{\sqrt{x+7}} + C$
(e) $-\frac{21}{\sqrt{x+7}} + C$

10. The series $\sum_{n=1}^{\infty} (-1)^n \frac{(n+2)!}{2^n n!}$

- (a) converges absolutely
- (b) converges conditionally
- (c) diverges by the comparison test
- (d) diverges by the ratio test
- (e) diverges by the root test

11. What is the coefficient of $(x+2)^2$ in the Taylor expansion of $f(x) = (x+3)^3$ about $a = -2$?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 6

12. Determine whether the series $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$ is convergent or divergent. If convergent, find the sum.

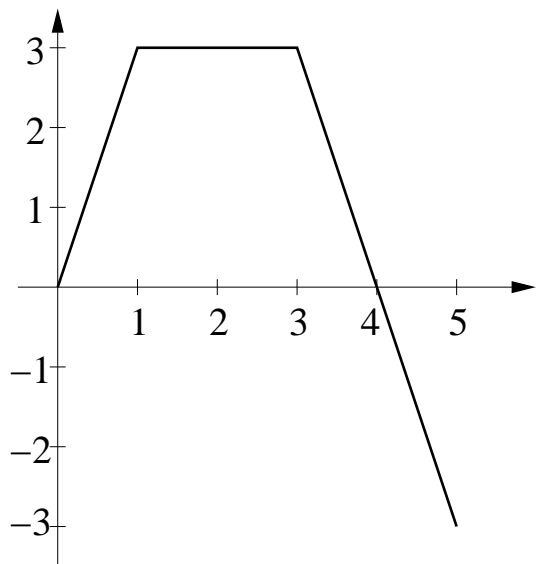
- (a) $\frac{5}{7}$ (b) $\frac{2}{7}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$ (e) diverges

13. The coefficient of x^4 in the power series for the function $f(x) = \frac{x^2}{1-2x}$ equals

- (a) -1 (b) 2 (c) -2 (d) 4 (e) -4

PART II (Calculators Allowed)

1. The graph of $y = f(x)$ is shown below. Estimate the value of the integral $\int_0^5 f(x) dx$.



- (a) 6 (b) 7.5 (c) 9 (d) 11.5 (e) 13.5
2. If $\int_2^8 f(x) dx = 6$, $\int_2^4 f(x) dx = 7$, and $\int_6^8 f(x) dx = 3$, find $\int_4^6 f(x) dx$.

- (a) 16 (b) 10 (c) 4 (d) 2 (e) -4

3. Express the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^4}$ as a definite integral and evaluate it.

- (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 1

4. Find the average value of the function $f(x) = 3x^2 - 2$ on the interval $[-1, 2]$.

- (a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) 1 (d) 1.5 (e) 3

5. Which of the following sequences converge?

I. $(-1)^n \frac{n}{1 + \sqrt{n}}$ II. $\frac{n-2}{3n+5}$ III. $\frac{\cos 3n}{2n-1}$

(a) I only (b) II only (c) III only (d) I and II only (e) II and III only

6. Find the length of the curve $y = 3 + 2x^{3/2}$, $0 \leq x \leq 1$.

(a) $\frac{2(10^{3/2} - 1)}{27}$ (b) $\frac{10^{3/2} - 1}{27}$ (c) $\frac{2(8^{3/2} - 1)}{27}$ (d) $\frac{2(7^{3/2} - 1)}{27}$ (e) $\frac{2(5^{3/2} - 1)}{27}$

7. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ II. $\sum_{n=1}^{\infty} \frac{2^n}{3e^n}$ III. $\sum_{n=1}^{\infty} \frac{n!}{4^n}$

(a) I only (b) II only (c) III only (d) I and II only (e) II and III only

8. Find the derivative of the function $\int_1^{x^2} \cos^2 t \, dt$.

(a) $2x \cos^2 x^2$ (b) $2x \cos^2 x$ (c) $-x^2 \cos^2 x^2$ (d) $-2x \cos^2 x$ (e) $-2x \cos^2 x^2$

9. A particle is moved along the x -axis by a force $F(x) = x^2 + 5x - 6$. How much work is done in moving the particle from $x = -2$ to $x = 4$?

- (a) 15 (b) 17 (c) 19 (d) 18 (e) 23

10. Suppose that $f'(x) = 2x^3 - \frac{8}{x^3}$ and $f(2) = 5$. Find a formula for $f(x)$ and then use it to evaluate $f(1)$.

- (a) $f(1) = \frac{5}{2}$ (b) $f(1) = 2$ (c) $f(1) = \frac{3}{2}$ (d) $f(1) = 1$ (e) $f(1) = \frac{1}{2}$

11. Find the x -coordinate of the centroid for the region bounded by $y = 0$, $x = 3$, and $y = 2x$.

- (a) $\frac{20}{9}$ (b) $\frac{5}{3}$ (c) 2 (d) $\frac{22}{9}$ (e) $\frac{7}{3}$

12. It is known that $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$. Use this formula to evaluate $\int \frac{3dx}{9 + 4x^2}$.

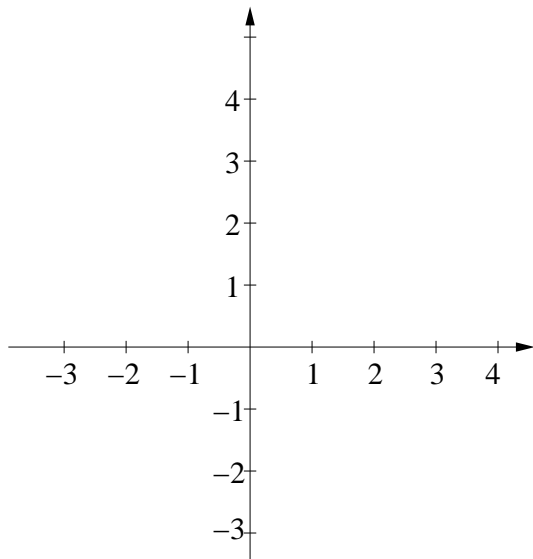
- (a) $\frac{1}{3} \tan^{-1} \frac{2x}{3} + C$
(b) $\frac{1}{2} \tan^{-1} \frac{2x}{3} + C$
(c) $\frac{1}{3} \tan^{-1} \frac{4x}{3} + C$
(d) $\frac{1}{2} \tan^{-1} \frac{4x}{3} + C$
(e) $\tan^{-1} \frac{2x}{3} + C$

PART III (Calculators Allowed)

1. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^n}{n3^n}$. Be sure to check for convergence or divergence the endpoints of the interval.

Answer:

2. Sketch the region enclosed by the curves $y = x^2$ and $y = 4x - x^2$. Then find the area A of this region using an appropriate integral.



Answer:

$A =$

3. (a) Find the partial fraction expansion of $R = \frac{3x + 2}{x^3 + x}$. Show your work.

Answer:

- (b) Use the answer in part (a) to evaluate $I = \int \frac{3x + 2}{x^3 + x} dx$. Show your work.

Answer:

4. (a) Use the trapezoidal rule with $n = 3$ to estimate the value of $I = \int_0^3 \frac{dx}{1+3x}$. Show your work.

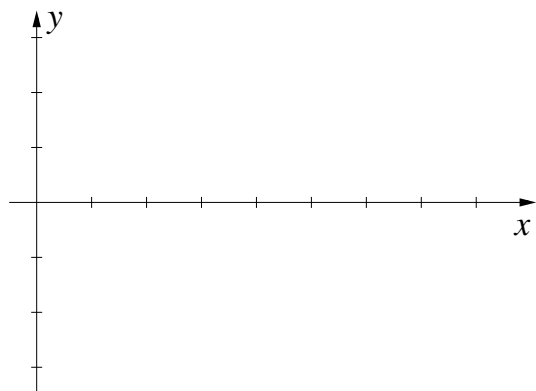
Answer:

- (b) The error formula for estimating the value of $\int_a^b f(x) dx$ using the trapezoidal rule is $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ where $K \geq |f''(x)|$ for all $a \leq x \leq b$. Use this formula to estimate the value of $|E_T|$ in part (a) using the smallest possible value of K . Be sure to show how you find K .

Answer:

Answer:

5. Find the volume V of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the x -axis. Sketch a graph of the region.



Answer:

ANSWER KEY FOR PART I

1. (c)
2. (c)
3. (b)
4. (a)
5. (d)
6. (a)
7. (d)
8. (e)
9. (b)
10. (a)
11. (c)
12. (d)
13. (d)

ANSWER KEY FOR PART II

1. (b)
2. (e)
3. (b)
4. (c)
5. (e)
6. (a)
7. (b)
8. (a)
9. (d)
10. (e)
11. (c)
12. (b)

ANSWER KEY FOR PART III

1. $-4 < x \leq 2$

2. $A = \frac{8}{3}$

3. (a) $R = \frac{3-2x}{x^2+1} + \frac{2}{x}$ (b) $I = 2 \ln x - \ln(x^2+1) + 3 \arctan x + C$

4. (a) $I \approx \frac{33}{35}$ (b) $K = 18, |E_T| \leq 4.5$ (c) $n \geq 64 > 45\sqrt{2}$

5. $V = \frac{\pi}{6}$