The braid indices of alternating links

Pengyu Liu

Preprint no. 2018-05

Abstract

It is well known in knot theory that any link can be represented by a closed braid and the braid index of a link is the invariant defined as the minimum number of strands in any closed braid representing the link. It is difficult in general to determine the braid index for a given link. However, in recent years, some progresses have been made due to the discovery of the HOMFLY polynomial. Yamada showed that the braid index of a link L equals the minimum number of Seifert circles of L. With this connection, one might conjecture that the braid index of an alternating link equals the number of Seifert circles in any of its reduced alternating diagrams. However, this is not true in general. In this dissertation, we prove this conjecture is in fact true for a class of alternating links. Specifically, we prove that if D is a reduced alternating diagram of an alternating link L, then the braid index b(L) equals the number of Seifert circles in D if and only if the Seifert graph of D contains no edge of weight one.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223