Spanning hypertrees, vertex tours

and meanders

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Abstract

This paper revisits the notion of a spanning hypertree of a hypermap introduced by one of its authors and shows that it allows to shed new light on a very diverse set of recent results. The tour of a map along one of its spanning trees used by Bernardi may be generalized to hypermaps and we show that it is equivalent to a dual tour described by Cori and Machì. We give a bijection between the spanning hypertrees of the reciprocal of the plane graph with 2 vertices and n parallel edges and the meanders of order n and a bijection of the same kind between semimeanders of order n and spanning hypertrees of the reciprocal of a plane graph with a single vertex and $n/2$ nested edges. We introduce hyperdeletions and hypercontractions in a hypermap which allow to count the spanning hypertrees of a hypermap recursively, and create a link with the computation of the Tutte polynomial of a graph. Having a particular interest in hypermaps which are reciprocals of maps, we generalize the reduction map introduced by Franz and Earnshaw to enumerate meanders to a reduction map that allows the enumeration of the spanning hypertrees of such hypermaps.

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