Spectral theory of Schrödinger type operator on spider graphs

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Abstract

We construct the definition of Brownian motion on the N-legged spider graph with infinite legs and Kirchhoff's gluing conditions at the origin and calculate the transition probability of this process. In addition, we study several important Markov moments, for instance the first exit time τ_L from the spider with the length L of all legs. The calculations give not only the moments of τ_L but also the distribution density for τ_L (all results of this section are new ones). For the spectral theory on the spider like quantum graphs, we start by constructing the spectral analysis on the finite interval of a three-legged spider graph and then pass it to infinity. Spectral analysis is performed for three different types of potentials. The fast-decreasing potentials, the fast-increasing potentials, mixed potentials, and its spectral theory. The details contain, the absolute continuous spectrum of multiplicity 3 and its construction using the reflection-transmission coefficients on each leg for the fast decreasing potential, Bohr's asymptotic formula for $N(\lambda)$ (the negative eigenvalues), instability of the discrete spectrum for the mixed potential on each leg of the spider graph. Furthermore, we have done symplectic analysis, and its representation on the spider quantum graph.

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