Catalan–Spitzer permutations

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Abstract

We study two classes of permutations intimately related to the visual proof of Spitzer's lemma and Huq's generalization of the Chung–Feller theorem. Both classes of permutations are counted by the Fuss–Catalan numbers. The study of one class leads to a generalization of results of Flajolet from continued fractions to continuants. The study of the other class leads to the discovery of a restricted variant of the Foata–Strehl group action.

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Key words and phrases: Fuss–Catalan number, Raney number, continued fraction, Motzkin path, Foata–Strehl group action.

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