1. Let $m^*$ denote the Lebesgue outer measure, and suppose that $E \subset \mathbb{R}$ satisfies $m^*(E) < \infty$. Prove that if $E$ is not measurable, then there exists an open set $U$ containing $E$ such that

$$m^*(U \setminus E) > m^*(U) - m^*(E).$$

2. Give an example of a sequence of measurable functions $\{f_n\}_{n\geq 1}$ on $\mathbb{R}$ such that

   (i) $\{f_n\}_{n\geq 1}$ converges to zero almost everywhere on $\mathbb{R}$, and

   (ii) for each open interval $A \subset \mathbb{R}$ with finite length, the sequence $\{f_n\}_{n\geq 1}$ *does not* converge uniformly on the complement $\mathbb{R} \setminus A$.

   (To receive full credit, you should prove that your example has these properties.)

3. Compute the following limit and justify your answer:

$$\lim_{n\to\infty} \int_0^\infty \left(1 + \frac{x}{n}\right)^{-n} \sin\left(\frac{x}{n}\right)\, dx.$$

4. Suppose that $f$ and $g$ are absolutely continuous functions on $[0, 1]$ and $g > 0$. Prove that the ratio $\frac{f}{g}$ is absolutely continuous on $[0, 1]$.