

THE ASYMPTOTIC APPROXIMATION OF THE
TRANSITION PROBABILITY FOR THE
RANDOM WALKS WITH HEAVY TAILS

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Abstract

The main result concerns the asymptotics, uniform in x and t , of the transition probability of a random walk with heavy tails. It is assumed that the jump distributions decay not too fast at infinity and possesses some form of “regular behavior”. The random walk is a Markov process and thus can be characterized in terms of their generators. We impose certain conditions on the Fourier transform of the kernel of the generator, which still allow us to consider rather general class of processes in \mathbb{Z}^d . The process we consider can be viewed as a generalization of the simple symmetric walk (in continuous time) for which both the central limit theorem and large deviation results are well-known. In terms of probability densities, these limit theorems give the asymptotics of $p(t, x, 0)$ when x is of order $t^{1/\alpha}$. For the class of random walk under consideration, we obtain the asymptotics of $p(t, x, 0)$ uniformly in t and x for all $t > 1$, $x \in \mathbb{R}^d$ covering, in particular, the regime of the central limit theorem and large deviations.