

ON THE STRUCTURE AND INVARIANTS OF  
CUBICAL COMPLEXES

Sarah J. Birdsong

Preprint no. 2013-05

**Abstract**

This dissertation introduces two new results for cubical complexes. The first is a simple statistic on noncrossing partitions that expresses each coordinate of the toric  $h$ -vector of a cubical complex, written in the basis of the Adin  $h$ -vector entries, as the total weight of all noncrossing partitions. This expression can then be used to obtain a simple combinatorial interpretation of the contribution of a cubical shelling component to the toric  $h$ -vector.

Secondly, a class of indecomposable permutations, bijectively equivalent to standard double occurrence words, may be used to encode one representative from each equivalence class of the shellings of the boundary of the hypercube. Finally, an adjacent transposition Gray code is constructed for this class of permutations, which can be implemented in constant amortized time.