Counting genus one partitions and permutations

Robert Cori and Gábor Hetyei

Preprint no. 2013-08

Abstract

We prove the conjecture by M. Yip stating that counting genus one partitions by the number of their elements and parts yields, up to a shift of indices, the same array of numbers as counting genus one rooted hypermonopoles. Our proof involves representing each genus one permutation by a four-colored noncrossing partition. This representation may be selected in a unique way for permutations containing no trivial cycles. The conclusion follows from a general generating function formula that holds for any class of permutations that is closed under the removal and reinsertion of trivial cycles. Our method also provides a new way to count rooted hypermonopoles of genus one, and puts the spotlight on a class of genus one permutations that is invariant under an obvious extension of the Kreweras duality map to genus one permutations.

2000 AMS Subject Classification: Primary 05C30; Secondary 05C10, 05C15

Key words and phrases: set partitions, noncrossing partitions, genus of a hypermap

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223