Symmetry, Reduction, and Constraints in Mechanics and Hydrodynamics

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Abstract

This dissertation has four objectives. First, to clarify the relationship between the seemingly different definitions of principal bundle appearing in the literature, with a particular emphasis on applications to theoretical mechanics, and to discuss these results in the context of reduction by symmetry. Second, to introduce and analyze a new system in the theory of nonholonomic mechanical systems with symmetry, the Chaplygin beanie — which is a synthesis of two simpler classical systems exhibiting geometric phase and nonholnomic constraints — and to exploit the coupling of nonholonomic constraints and symmetry to generate locomotion and achieve an asymptotic form of underactuated control. Third, to introduce the constrained N-vortex problem as a low-order model for certain problems in ideal hydrodynamics, and to analyze this problem using Dirac's method for constrained Hamiltonian systems. Finally, the fourth goal is to concisely synthesize and carefully present basic concepts from geometric mechanics that are scattered throughout the literature, to provide geometric motivation to constructions that are defined in the literature merely by formulas, and to emphasize coordinate-free definitions for their many benefits while at the same time computing examples in coordinates to aid in the understanding of concrete calculations.

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