Path intergal methods using Feynman-Kac formula and reflecting Brownian motions for Neumann and Robin Problems

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Abstract

In this dissertation, we propose numerical methods for computing the boundary local time of reflecting Brownian motion (RBM) in 3d and its use in the probabilistic representation of the solution of the Laplace equation with the Neumann and Robin boundary conditions respectively. Approximations of RBM based on a walk-on-spheres (WOS) and random walk on lattices are discussed and tested for sampling RBM paths and their applicability in finding accurate approximation of the local time and discretization of the probabilistic formula. Numerical tests for a cube domain have shown the convergence of the numerical methods as the time length of RBM paths and number of paths sampled increases. Spherical, ellipsoidal, nonconvex domains were also tested to prove the efficiency and accuracy of the algorithm. Moreover, an exterior Neumann problem of a many-spheres system further demonstrated the effectiveness of the method even the starting point of the path lying exactly on the boundary. Additionally, the application in electrical impedance tomography to solve the forward problem further demonstrates the simplicity and efficiency of our approach, which is extremely important for some reconstruction methods of the inverse problem. Other applications in material science in calculating the electrical properties of materials in special shapes are also discussed as possible future work.

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