High order $\mathcal{H}(\mathbf{div})$ discontinuous Galerkin methods for MHD equations

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Preprint no. 2016-12

Abstract

In this dissertation, we investigate the divergence-free discontinuous Galerkin method using the $\mathcal{H}(\mathbf{div})$ basis to solve the nonlinear ideal magnetohydrodynamics (MHD) equations. This is a novel approach to ensure the divergence-free condition on the magnetic field. The idea is to add on each element extra bubble functions from the same order hierarchical $\mathcal{H}(\mathbf{div})$-conforming basis to reduce the higher order divergence, and then extra linear edge functions to remove the linear term of divergence. As a consequence, this method has a smaller computational cost than the traditional discontinuous Galerkin method with standard piecewise polynomial spaces. We formulate the discontinuous Galerkin method using our $\mathcal{H}(\mathbf{div})$-conforming basis and perform extensive two-dimensional numerical experiments for both smooth solutions and solutions with discontinuities. Our computational results show that the global divergence is largely reduced, but with a relatively small cost on the accuracy of the solution spaces.

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