

INTEGRABILITY OF THE NOSÉ–HOOVER EQUATION

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Abstract

In this work we consider the Nosé–Hoover equation for a one dimensional oscillator

$$\dot{x} = -y - xz, \quad \dot{y} = x, \quad \dot{z} = \alpha(x^2 - 1).$$

It models the interaction of a particle with a heat–bath. We contribute to the understanding of its global dynamics, or more precisely, to the topological structure of its orbits by studying the integrability problem. We prove that $\alpha = 0$ is the only value of the parameter for which the system is integrable, and in this case we provide an explicit expression for its first integrals.

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