Asymptotic analysis of the Anderson parabolic problem and the Moser's type optimal stopping problem

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Abstract

The central objects of the thesis are the Anderson parabolic problem and the Moser's type optimal stopping problem:

(1) In the lattice parabolic Anderson problem, we study the quenched and annealed asymptotics for the solutions of the lattice parabolic Anderson equation in the situation in which the underlying random walk has long jumps and belongs to the domain of attraction of the stable process.

The i.i.d random potential in our case is unbounded from above with regular Weibull type tails. Similar models but with the local basic Hamiltonian (lattice Laplacian) were analyzed in the very first work on intermittency for the parabolic Anderson problem by J. Gärtner and S. Molchanov. We will show that the long range model demonstrates the new effect. The annealed (moment) and quenched (almost sure) asymptotics of the solution have the same order in contrast to the case of the local models for which these orders are essentially different.

(2) Concerning Moser's problem, we study two related optimization problems for i.i.d. random variables X_i, $i = 1, 2, \ldots, n\}$, referred to as the generalized Moser's problem: a) Find $\max_{\tau \leq n} EX_\tau$ ($\tau \leq n$ are the stopping times). b) Find $\tau : P\{X_\tau = M_n\} = \max$, here $M_n = \max_{0 \leq i \leq n} X_i$. For the wide class of continuous distribution functions $F_X(x)$ with regular tails, we will present the asymptotic formulas for the optimal thresholds and analyze the relationship between the Moser's type problem and the classical secretary problem with information. The present paper is structured as follows: The first two chapters contain preliminary information.

In Chapter 1, we summarize some important properties and results about slowly varying functions.

In Chapter 2, we introduce the Anderson parabolic model, summarize some main results, such as the uniqueness and existence and the asymptotic properties of the solution $u(t, x)$, for the parabolic Anderson model on Z^d and R^d with homogenous potentials, and discuss some limit theorems for random walks with heavy-tailed long jumps.

In Chapter 3, we prove several results on the annealed and quenched behavior of $u(t, x)$ as $t \to$ with Weibull's potential. We will show that the long range model demonstrates the new effect. The annealed (moment) and quenched (almost sure) asymptotics of the solution have the same order in contrast to the case of the local models for which these orders are essentially different.

In Chapter 4, we study Moser's problem, present the asymptotic formulas for the optimal thresholds of the wide class of continuous distribution functions $F_X(x)$ with regular tails, and analyze the relationship between the Moser's type problem and the classical secretary problem with information.

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