MATH 7120/8120: Probability Theory I

Syllabus

Book: Probability and Measure, Anniversary Edition by Patrick Billingsley

- Chapter 1: Probability
  Sections 2-4
  –Probability spaces, σ-fields, properties of probability
  –π-λ Theorem, uniqueness
  –Independence of classes of sets, Borel-Cantelli Lemmas, Kolmogorov's Zero-One Law

- Chapters 2: Measure
  Sections 10,13
  –General measure spaces, σ-finite measures, uniqueness
  –Measurable functions, random variables, and random vectors
  –Approximation by simple functions, transforms of measures

- Chapter 3: Integration
  Sections 15,16,18
  –Construction of the Lebesgue integral and basic properties
  –Change of variables, monotone and dominated convergence, product measures and Fubini's Theorem

- Chapter 4: Random Variables and Expected Values
  Sections 20-22
  –Independence of random variables, properties of cumulative distribution functions, multivariate distributions
  –Convergence of random variables in distribution, in probability, and almost surely
  –Expectation as a Lebesgue integral, expectation of functions of random vectors, properties of expectation, moment generating functions
  –Sums of random variables, weak and strong laws of large numbers

- Chapter 5: Convergence of Distributions
  Sections 14, 25-27
  –Relation to convergence in probability and almost surely
  –Convergence to a point mass, Slutsky's Theorem
  –Characteristic functions: basic properties.
  –Central limit theorem for iid random variables with a finite second moment

- Chapter 6: Derivatives and Conditional Probability
  Sections 32-34
  –Absolutely continuous, singular, and equivalent measures
  –Densities and the Radon-Nikodym Theorem

–Conditioning on sets with positive probability
–Conditioning on σ-fields
–Basic properties of conditional probability and conditional expectation
–Relation between conditional probability and conditional expectation
–Tower property of conditional expectation
–Conditional distributions