

ON THE STRUCTURE OF THE  $d$ -INDIVISIBLE  
NONCROSSING PARTITION POSETS

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**Abstract**

We study the poset of  $d$ -indivisible noncrossing partitions introduced by Mühle, Nadeau and Williams. These are noncrossing partitions such that each block has cardinality 1 modulo  $d$  and each block of the dual partition also has cardinality 1 modulo  $d$ . Generalizing the work of Speicher, we introduce a generating function approach to reach new enumerative results and recover some known formulas on the cardinality, the Möbius function and the rank numbers. We compute the antipode of the Hopf algebra of  $d$ -indivisible noncrossing partition posets. Generalizing work of Stanley, we give an edge labeling such that the labels of the maximal chains are exactly the  $d$ -parking functions. This edge labeling induces an  $EL$ -labeling. We also introduce  $d$ -parking trees which are in bijective correspondence with the maximal chains.

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