Discrete maximum principle preserving scheme for 1-d nonlocal to local diffusion problem: development, analysis, simulation, and application

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Abstract

Diffusion is a scientific phenomena that can be modeled by partial differential equations. In this paper we first explore the development of equations for local, nonlocal, and quasi-nonlocal diffusion. Methods of finding solutions will be discussed as well as the properties of each diffusion model type. These properties include satisfying the maximum principle and demonstrating the well-posedness of each model which is through the solutions existence, uniqueness, and stability.

Also in a recent paper, a quasi-nonlocal coupling method was introduced to seamlessly bridge a nonlocal diffusion model with the classical local diffusion counterpart in a one-dimensional space. The proposed coupling framework removes interfacial inconsistency, preserves the balance of fluxes, and satisfies the maximum principle of the diffusion problem. However, the numerical scheme proposed in that paper does not maintain all of these properties on a discrete level. We resolve this issue by proposing a new finite difference scheme that ensures the balance of fluxes and the discrete maximum principle. We rigorously prove these results and provide the stability and convergence analyses accordingly. In addition, we provide the Courant-Friedrichs-Lewy (CFL) condition for the new scheme and test a series of benchmark examples which confirm the theoretical findings.

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