MATH 1241 – CALCULUS I SPRING 2022

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: Last Name

PRINT: First Name

PRINT: Student ID #

PRINT: Instructor

PRINT: Section

This exam is divided into three parts. **Calculators are not allowed on Part I and during the first hour of the exam**. You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the exam during the first hour, but you cannot use your calculator during this time. You may use your calculator only after you have submitted Part 1 and the exam proctor has announced that calculators are allowed.

PART I

- Part I consists of 15 multiple choice problems. These problems must be answered without the use of a calculator.
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer make sure that you completely erase your old answer and any extraneous marks. You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "A" to indicate Part I.
- You must hand in the test booklet and bubble sheet for Part I exactly one hours after the exam started.
- Only scratch paper provided by the proctor may be used.

Part I: Multiple Choice, No Calculators

1. Let $f(x)=2x^4+2x-7$. Evaluate $f'(1)$.

   (a) -3

   (b) 4

   (c) 3

   (d) 10

   (e) 13

2. Let $g(x)=\dfrac{10}{\sqrt{x}}$. Find $g'(x)$.

   (a) $-5x^{-\frac{3}{2}}$

   (b) $5x^{\frac{3}{2}}$

   (c) $5\sqrt{x}$

   (d) $-10x^{-\frac{3}{2}}$

   (e) $\frac{5}{3}\sqrt{x}$

3. Let $h(x)=\sqrt{3x^2-3}$. Find $h'(2)$.

   (a) $\frac{1}{6}$

   (b) $\frac{1}{3}$

   (c) 2

   (d) 3

   (e) 5

4. Let $f(x)=\ln\left(x^3(4x+1)^5\right)$. Find $f'(1)$.

   (a) 0

   (b) 1

   (c) 3

   (d) 5

   (e) 7

Part I: Multiple Choice, No Calculators

5. Let $K(x) = 3 + 3x^2 + e^{2x-4}$. Find $K'(2)$.

(a) $2e$

(b) $4e$

(c) 12

(d) 14

(e) 18

6. Find the equation of the tangent line to the graph of $f(x) = \dfrac{5x-2}{x^2}$ at the point where $x = 1$.

(a) $y = x - 4$

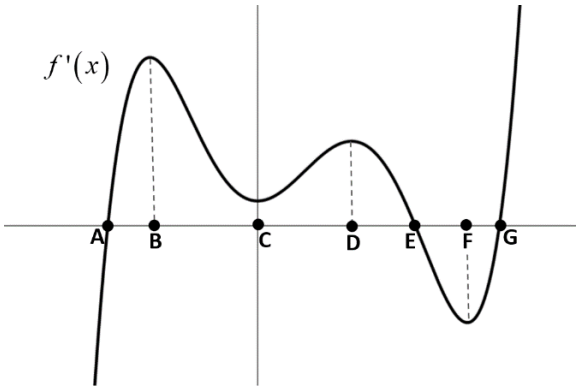
(b) $y = -x + 4$

(c) $y = x + 2$

(d) $y = -x + 2$

(e) $y = 2x - 4$

7. The graph of $f'(x)$ is shown below. Give the all interval(s) where the function $f(x)$ increases.



(a) $(A,E) \cup (G,\infty)$

(b) $(B,C) \cup (D,F)$

(c) $(B,C) \cup (D,E)$

(d) $(B,D) \cup (G,\infty)$

(e) $(-\infty,B) \cup (C,D) \cup (F,\infty)$

8. Where does the graph of $f(x) = 2x^3 - 15x^2 + 24x + 6$ have a relative minimum?

(a) $x = -4$

(b) $x = -1$

(c) $x = 0$

(d) $x = 1$

Part I: Multiple Choice, No Calculators

(e) $x = 4$

9. Evaluate $\lim_{x\to 3} \dfrac{x^2+2x-15}{x-3}$.

(a) 3

(b) 5

(c) 8

(d) 15

(e) The limit does not exist

10. Evaluate $\lim_{x\to \pi} \dfrac{\cos x}{1-\sin x}$.

(a) $-\pi$

(b) -1

(c) 0

(d) 1

(e) The limit does not exist

11. Evaluate $\lim_{x\to \infty} \dfrac{2x-12x^3}{3x^3+2x^2-4x}$.

(a) -4

(b) $\frac{2}{3}$

(c) 3

(d) 6

(e) The limit does not exist

12. Calculate $\frac{d}{dx}\left[x^2 \tan x\right]$.

(a) $2x\sec^2 x$

(b) $2x+\sec^2 x$

(c) $2x\tan x - x^2\sec^2 x$

(d) $2x\tan x - x^2\sec^2 x$

(e) $2x\tan x + x^2\sec^2 x$

Part I: Multiple Choice, No Calculators

13. Let $y=(3x+1)^2$. Evaluate the differential dy when $x=2$ and $dx=0.5$.

(a) 20

(b) 20.5

(c) 21

(d) 21.5

(e) 22

14. Find the general antiderivative of $g(x)=12x^3+\sqrt{x}-\frac{6}{x^3}$.

(a) $12x^4+\frac{1}{2}x^{\frac{1}{2}}-\frac{6}{x^4}+C$

(b) $3x^4+\frac{1}{2}x^{\frac{1}{2}}-\frac{3}{x^4}+C$

(c) $3x^4+\frac{1}{2}x^{\frac{1}{2}}+\frac{3}{x^4}+C$

(d) $3x^4+\frac{2}{3}x^{\frac{3}{2}}+\frac{3}{x^2}+C$

(e) $3x^4+\frac{2}{3}x^{\frac{3}{2}}-\frac{3}{x^2}+C$

15. The derivative of the function $f(x)=\frac{x^2+8x-2}{\sqrt{x}}$ is

(a) $f'(x)=\frac{1}{2}x^{\frac{1}{2}}-2x^{-\frac{1}{2}}+2x^{-\frac{3}{2}}$

(b) $f'(x)=\frac{3}{2}x^{\frac{1}{2}}-2x^{-\frac{1}{2}}-x^{-\frac{3}{2}}$

(c) $f'(x)=\frac{3}{2}x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+x^{-\frac{3}{2}}$

(d) $f'(x)=\frac{2x+8}{\sqrt{x}}$

(e) $f'(x)=\frac{2x+8}{\frac{1}{2}x^{-\frac{1}{2}}}$

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This exam is divided into three parts. **Calculators are allowed on Part II, but only after Part I has been handed in and the proctor has announced that calculators may be used.** You have 3 hours for the entire exam – you may hand in Part II of the exam at the end of the exam along with Part III.

PART II

- Part II consists of 13 multiple choice problems. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer make sure that you completely erase your old answer and any extraneous marks. You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "B" to indicate Part II.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

Part II: Multiple Choice, Calculators Allowed

1. Suppose that the derivative of f is given by $f'(x) = 2x + \frac{1}{x}$ for $x > 0$ and that $f(1) = 2$. Find a formula for $f(x)$.

   (a) $x^2 + \frac{1}{x^2} + 2$

   (b) $x^2 - \frac{1}{x^2} + 1$

   (c) $x^2 + \ln|x| + 2$

   (d) $x^2 + \ln|x| - 1$

   (e) $x^2 + \ln|x| + 1$

2. Let $f(x) = e^{x+1} + 1$. Then $f^{-1}(x)$, the inverse of $f(x)$, is equal to

   (a) $\ln(x-2)$

   (b) $\ln(x-1) + 1$

   (c) $\ln(x-1) - 1$

   (d) $\ln x - 2$

   (e) f is not invertible

3. Let $f(x) = \begin{cases} cx^2 - 3x + 5, & x < 2 \\ 2x + 3, & x \geq 2 \end{cases}$. For what value of c is f continuous?

   (a) 1

   (b) 2

   (c) 3

   (d) 4

   (e) 5

4. Suppose f^{-1} is the inverse function of a differentiable function f, $f(2) = 7$ and $f'(2) = \frac{1}{6}$. Then $\left(f^{-1}\right)'(7) =$

   (a) 6

   (b) $\frac{1}{6}$

   (c) $\frac{7}{2}$

   (d) $\frac{2}{7}$

   (e) 7

Part II: Multiple Choice, Calculators Allowed

5. A particle moves along a path given by $h(t) = t^3 - 2t^2 + 8t + 4$, where t is measured in seconds and h is measured in feet. What is the particle's acceleration when $t = 3$?

(a) $-3\ ft/sec^2$

(b) $3\ ft/sec^2$

(c) $9\ ft/sec^2$

(d) $14\ ft/sec^2$

(e) $18\ ft/sec^2$

6. Find the linear approximation for the function $f(x) = 2 + x + \cos(2x)$ at $a = 0$.

(a) $L(x) = 2x - 1$

(b) $L(x) = 3x + 1$

(c) $L(x) = x - 3$

(d) $L(x) = x + 3$

(e) $L(x) = x + 2$

7. Let $f(x) = 3x^2 + x - 2$. Evaluate $\lim_{h \to 0} \dfrac{f(5+h) - f(5)}{h}$.

(a) 12

(b) 18

(c) 21

(d) 31

(e) 42

8. The **first derivative** of a function f is given by $f'(x) = 2(x-3)^2(x-1)(x+2)$. Which one of the following statements is correct?

(a) f has a relative maximum at $x = -2$

(b) f is increasing on the interval $(-2, 1)$

(c) f has a relative maximum at $x = 1$

(d) f is decreasing on the interval $(-\infty, -2)$

(e) f has a relative maximum at $x = 3$

Part II: Multiple Choice, Calculators Allowed

9. Suppose that $5x-3xy^3=y+2$. Find the derivative $\frac{dy}{dx}$ at the point $(-1,1)$.

(a) -4

(b) $-\frac{4}{5}$

(c) $-\frac{1}{4}$

(d) $\frac{1}{4}$

(e) $\frac{4}{5}$

10. Let $f(x)=kx+\frac{2}{x}$ for some constant k. This function has a relative minimum at $x=1$. Find the value of k.

(a) $k=-2$

(b) $k=-1$

(c) $k=0$

(d) $k=1$

(e) $k=2$

11. We wish to solve $x^3-3x=4$ using Newton's method. Use $x_1=2$ as your initial approximation and find x_2, the next approximation. (You are not being asked for the exact solution.) Round your answer to three decimal places.

(a) 2.196

(b) 2.222

(c) 2.302

(d) 2.381

(e) 2.411

12. Use the Mean Value Theorem to find the greatest function value that $f(4)$ can be given that $f(x)$ is differentiable (and hence continuous) everywhere, $f(1)=2$ and $f'(x)\le 2$ for all x.

(a) 2

(b) 4

(c) 6

(d) 8

(e) 10

Part II: Multiple Choice, Calculators Allowed

13. The derivative of the function $f(x) = \arcsin\left(x^3 + 5\right)$ is

(a) $f'(x) = \dfrac{3x^2}{\sqrt{1-\left(x^3+5\right)^2}}$

(b) $f'(x) = \dfrac{3x^2}{\sqrt{1-x^2}}$

(c) $f'(x) = \dfrac{x^3+5}{\sqrt{1-\left(3x^2\right)^2}}$

(d) $f'(x) = 3x^2 \arccos\left(x^3+5\right)$

(e) $f'(x) = \arccos\left(x^3+5\right)$

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PART III

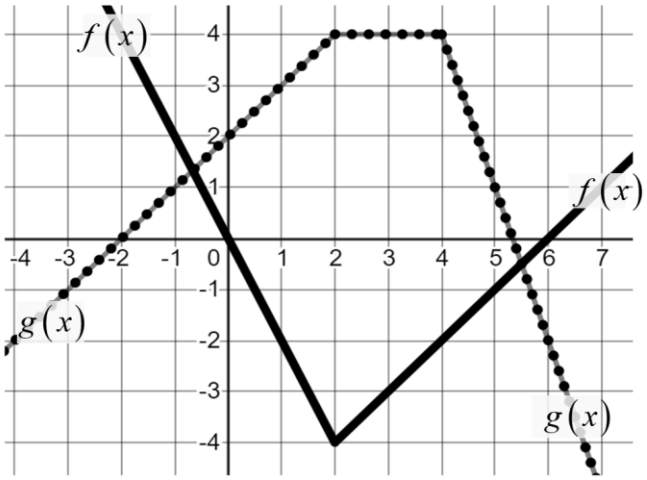
- Part III consists of 5 free response problems. Each one of these problems is worth 8 points. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question you are required to show complete and detailed justification of your answer by clearly and neatly showing all your work.
- Final answers (correct or incorrect) without supporting work will not receive any credit. Conversely, incorrect final answers that are supported by correct and clearly shown processes will receive partial credit. It is important that you SHOW ALL YOUR WORK, neatly and clearly.
- Work that is illegible or disorganized will not be graded.
- All work must be shown in the space provided. You may use the blank back of each page as scratch paper, but work done on these pages or other scratch paper will not be considered.
- Please provide final answers in the final answer boxes. (Please do not include any other work or markings in the final answer boxes.)
- Make sure you print the required information (name/ID number) on EVERY page of the test booklet.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

MATH 1241
Common Final Exam
Spring 2022

Part III: Free Response
Calculators Allowed
Work must be shown to receive credit

PRINT: Last, First

1. Functions f and g are given by the following graphs. Give each answer in its exact form.



8 POINTS

Show work in this space

a. Let $H(x) = f(x) \cdot (2x+2)$. Evaluate $H'(-1)$

Final Answer

b. Let $P(x) = \dfrac{2x^2}{g(x)}$. Evaluate $P'(3)$.

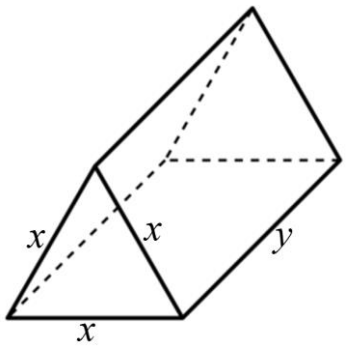
Final Answer

c. Let $K(x) = f(g(x))$. Evaluate $K'(5)$.

Final Answer

PRINT: Last, First

2. A popular European chocolate bar is sold in triangular-prism-shaped box, where the ends are equilateral triangles with sides x, and the box has a length of y (see the diagram below). The box holds a volume of 54 cubic inches. The configuration of the box as it is being sold right now does not optimize the amount of packaging material used. If we were to redesign this box in order to minimizes the amount of packaging material needed (i.e. minimize the surface area of the box), what would the dimensions x and y be? *If needed, round your answer to 3 decimal places, or leave it as an exact value.* Hint: The area of an equilateral triangle with sides x is given by $\frac{\sqrt{3}}{4}x^2$



8 POINTS

Show work in this space

Final Answer

$x =$

Final Answer

$y =$

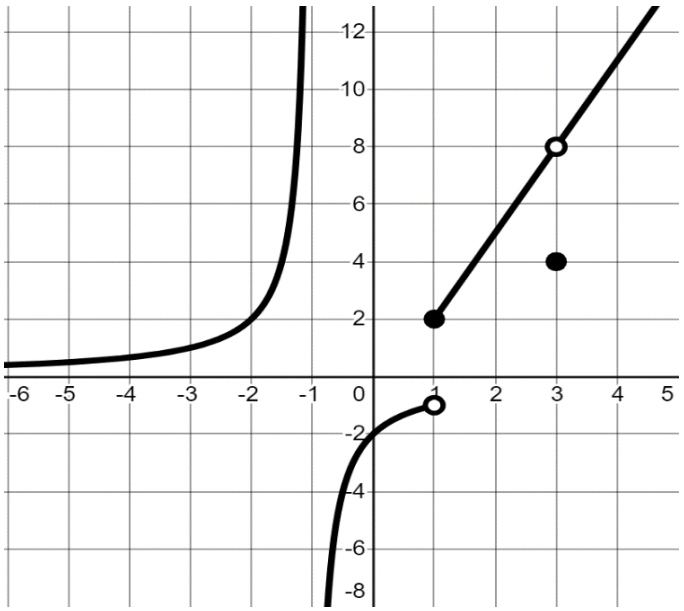
PRINT: Last, First

3. The graph of $y = f(x)$ is as shown.

Find each of the flowing limits.

- If the limits exist, give its value.
- For limits that approach positive or negative infinity, enter ∞ or $-\infty$.
- If the limit does not exsist, state your answer as DNE.

8 POINTS



Show work in this space

a. Find $\lim_{x \to -1^-} f(x) =$

b. Find $\lim_{x \to -1} f(x) =$

c. Find $\lim_{x \to 1^+} f(x) =$

d. Find $\lim_{x \to 1} f(x) =$

e. Find $\lim_{x \to 3} f(x) =$

f. Find $f(3) =$

4. Calculate the following limits using L'Hopitals rule.
Leave all answers as exact values. 8 POINTS

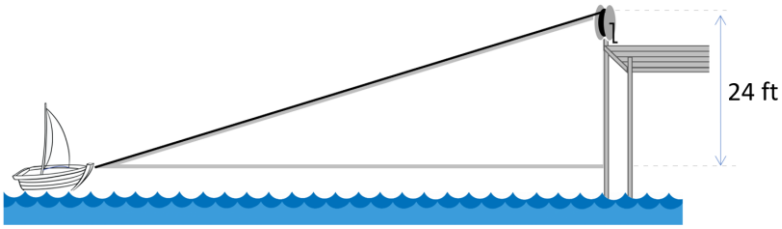
Show work in this space

a. $\lim_{x\to 0} \frac{e^x - x - 1}{4x^2}$

b. $\lim_{x\to \infty} x^2 e^{-x^2}$

c. $\lim_{x\to \infty} \left(1+\frac{5}{x}\right)^{8x} =$

5. A boat is pulled into a dock by means of a rope attached to a pulley on the dock. The rope is attached to the front of the boat, which is 24 feet below the level of the pulley. If the rope is pulled through the pulley at a rate of 27 ft/min, at what rate will the boat be approaching the dock when 30 ft of rope is out? If needed, round your answer to 3 decimal places, or leave it as an exact value.



8 POINTS

Show work in this space

Final Answer