MATH 1242 – CALCULUS II SPRING 2022

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: Last Name

PRINT: First Name

PRINT: Student ID #

PRINT: Instructor

PRINT: Section

This exam is divided into three parts. **Calculators are not allowed on Part I and during the first hour of the exam**. You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the exam during the first hour, but you cannot use your calculator during this time. You may use your calculator only after you have submitted Part 1 and the exam proctor has announced that calculators are allowed.

PART I

- Part I consists of 15 multiple choice problems. These problems must be answered without the use of a calculator.
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer make sure that you completely erase your old answer and any extraneous marks. You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "A" to indicate Part I.
- You must hand in the test booklet and bubble sheet for Part I exactly one hours after the exam started.
- Only scratch paper provided by the proctor may be used.

Part I: Multiple Choice, No Calculators

1. Evaluate the integral $\int 3x^2 - \frac{4}{x} - 4\sin x \, dx$.

(a) $x^3 - 4\ln|x| - 4\cos x + C$

(b) $x^3 - 4\ln|x| + 4\cos x + C$

(c) $x^3 + 4\ln|x| - 4\cos x + C$

(d) $x^3 - \frac{4}{x^2} - 4\cos x + C$

(e) $x^3 - \frac{4}{x^2} - 4\sin^2 x + C$

2. Evaluate the integral $\int_1^3 2x + 3 \, dx$.

(a) 4

(b) 9

(c) 12

(d) 14

(e) 18

3. Evaluate the integral $\int x\sqrt{1-x^2} \, dx$.

(a) $x^{\frac{3}{2}} - x + C$

(b) $\left(1-x^2\right)^{\frac{1}{2}} - \frac{1}{2}x^2\left(1-x^2\right)^{-\frac{1}{2}} + C$

(c) $-\frac{1}{3}\left(1-x^2\right)^{\frac{3}{2}} + C$

(d) $-\frac{x}{3}\left(1-x^2\right)^{\frac{3}{2}} + C$

(e) $\frac{x^2}{2}\left(x - \frac{x^3}{3}\right)^{\frac{3}{2}} + C$

Part I: Multiple Choice, No Calculators

4. Evaluate the integral $\int_0^{2\pi} \cos\theta \, d\theta$.

(a) $-\frac{\pi}{2}$

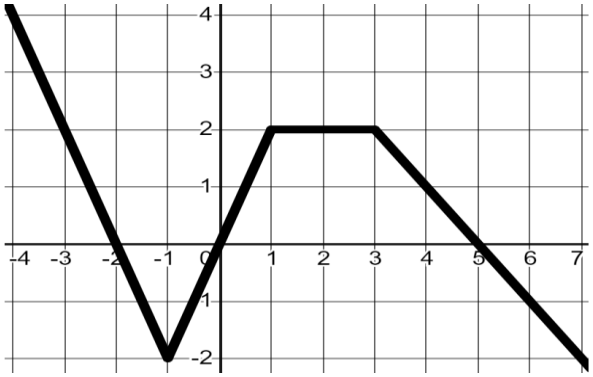
(b) 0

(c) $\frac{\pi}{2}$

(d) 2

(e) 2π

5. Calculate $\int_{-4}^{3} f(x)\,dx$ if the graph of $f(x)$ is



(a) –11

(b) –5

(c) 5

(d) 7

(e) 9

6. Compute $f''(x)$ where $f(x) = \int_0^{\sin x} e^t \, dt$. (Find the second derivative.)

(a) $e^{\sin x} \cdot \cos^2 x - e^{\sin x} \cdot \sin x$

(b) $e^{\sin x} \cdot \cos^2 x + e^{\sin x} \cdot \sin x$

(c) $e^{\sin x} \cdot \cos x - e^{\sin x} \cdot \sin^2 x$

(d) $e^{\sin x} \cdot \cos x + e^{\sin x} \cdot \sin^2 x$

(e) $e^{\sin x} \cdot \cos^2 x - e^{\sin x} \cdot \sin^2 x$

Part I: Multiple Choice, No Calculators

7. The limit $\lim_{n\to\infty}\sum_{i=1}^{n}\ln\left(1+\frac{2i}{n}\right)\cdot\frac{2}{n}$ equals the definite integral.

(a) $\int_{1}^{3}\ln(1+x)\,dx$

(b) $\int_{1}^{2}\ln(x)\,dx$

(c) $\int_{1}^{3}\ln(x)\,dx$

(d) $\int_{1}^{2}\ln(1+x)\,dx$

(e) $\int_{2}^{3}\ln(1+x)\,dx$

8. The sequence $\{a_n\}$, where $a_n=\frac{8n^2-2n}{4n-2n^2}$

(a) Converges to -4

(b) Converges to -2

(c) Converges to 0

(d) Converges to 2

(e) Diverges

9. Find the value of B in the partial fraction $\frac{4x-9}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$.

(a) $B=-9$

(b) $B=5$

(c) $B=-5$

(d) $B=1$

(e) $B=-1$

Part I: Multiple Choice, No Calculators

10. Compute the integral $\int_0^2 xe^x dx$.

(a) $2e^4$

(b) e^2+1

(c) $\frac{1}{2}e^4-\frac{1}{2}$

(d) $\frac{1}{2}e^4$

(e) $\frac{1}{2}e^2-\frac{1}{2}$

11. Find the Taylor expansion for $f(x)=e^{2x}$ about $a=1$.

(a) $1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+\cdots$

(b) $2e^2+2e^2(x-1)+\frac{2e^2(x-1)^2}{2!}+\frac{2e^2(x-1)^3}{3!}+\cdots$

(c) $1+2e^2+\frac{4ex^2}{2!}+\frac{8ex^2}{3!}+\cdots$

(d) $e^2+2e^2(x-1)+\frac{4e^2(x-1)^2}{2!}+\frac{8e^2(x-1)^3}{3!}+\cdots$

(e) $e^2+1+(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}+\cdots$

12. Which of the following statements is/are correct?

I. $\sum_{n=0}^{\infty}\left(\frac{5}{4}\right)^n$ diverges II. $\sum_{n=1}^{\infty}\frac{\sin^2 n}{n^3}$ converges III. $\sum_{n=1}^{\infty}(-1)^n\frac{3n}{2n+1}$ converges

(a) I. and II. only

(b) II. and III. only

(c) I., II. and III.

(d) II. only

(e) III. only

Part I: Multiple Choice, No Calculators

13. Since the repeating decimal $0.383838\ldots$ can be represented as the geometric series $\frac{38}{100}+\frac{38}{100^2}+\frac{38}{100^3}+\cdots$, the value of this repeating decimal is

(a) $\frac{62}{99}$

(b) $\frac{100}{62}$

(c) $\frac{38}{62}$

(d) $\frac{38}{99}$

(e) $\frac{38}{100}$

14. Evaluate the integral $\int_2^5 \frac{1}{x-2}dx$.

(a) Converges to $\frac{1}{5}$

(b) Converges to $\frac{1}{2}$

(c) Converges to $\ln 3$

(d) Diverges to ∞

(e) Diverges to $-\infty$

15. Evaluate the integral $\int_2^\infty x^2 e^{8-x^3}dx$.

(a) Converges to $\frac{1}{3}$

(b) Converges to 0

(c) Converges to $\frac{1}{3e^3}$

(d) Converges to 3

(e) Diverges to ∞

MATH 1242 – CALCULUS II SPRING 2022

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: Last Name

PRINT: First Name

PRINT: Student ID #

PRINT: Instructor

PRINT: Section

This exam is divided into three parts. **Calculators are allowed on Part II, but only after Part I has been handed in and the proctor has announced that calculators may be used.** You have 3 hours for the entire exam – you may hand in Part II of the exam at the end of the exam along with Part III.

PART II

- Part II consists of 12 multiple choice problems. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer make sure that you completely erase your old answer and any extraneous marks. You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "B" to indicate Part II.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

Part II: Multiple Choice, Calculators Allowed

1. Select the correct table formula from the list

$$\int \sqrt{u^2-a^2}\,du = \tfrac{1}{2}u\sqrt{u^2-a^2} - a^2\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}\left(2u^2-a^2\right)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{|u|} + C$$

and use it to compute $\int x\sqrt{x^4-64}\,dx$.

(a) $\dfrac{x}{8}\left(2x^4-64\right)\sqrt{x^4-64} - \dfrac{4096}{8}\ln\left|x+\sqrt{x^4-64}\right| + C$

(b) $\dfrac{x^2}{4}\left(2x^4-64\right)\sqrt{x^4-64} - \dfrac{4096}{4}\ln\left|x^2+\sqrt{x^4-64}\right| + C$

(c) $\tfrac{1}{4}x^2\sqrt{x^4-64} - 32\ln\left|x^2+\sqrt{x^4-64}\right| + C$

(d) $\tfrac{1}{2}x^2\sqrt{x^4-64} - 64\ln\left|x^2+\sqrt{x^4-64}\right| + C$

(e) $\sqrt{x^4-64} - 8\cos^{-1}\dfrac{8}{|x|} + C$

2. Find the area of the region bound by the curves $y = 5-x^2$ and $y = 1$ is

(a) $\dfrac{20}{3}$

(b) $\dfrac{22}{3}$

(c) $\dfrac{25}{3}$

(d) $\dfrac{28}{3}$

(e) $\dfrac{32}{3}$

Part II: Multiple Choice, Calculators Allowed

3. Find the average value of the function $f(x) = 6x^2 - 1$ on the interval $[-1, 3]$.

(a) 2

(b) 6

(c) 13

(d) 26

(e) 52

4. Given the table of values for a function $f(x)$, Use Simpson's rule with 4 subintervals of equal width to approximate $\int_0^{12} f(x)\,dx$.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	2	4	2	5	6	1	0	3	1	3	1	4	2

(a) 9

(b) 12

(c) 24

(d) 30

(e) 36

5. Given that the total mass is $\frac{32}{3}\rho$, calculate the x-coordinate of the centroid (center of mass) of the region bounded by $y = 0$ and $y = -x^2 + 4x$. (Hint: graph the region.)

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 2

(d) 4

(e) 8

Part II: Multiple Choice, Calculators Allowed

6. Which of the following is the length of the curve $y = \frac{1}{x}$ over the interval $[1, 2]$?

(a) 0.685

(b) 0.823

(c) 0.922

(d) 1.222

(e) 1.132

7. Which of the following series converges conditionally, but not absolutely?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+2}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+n+3)}{4n^2+1}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n+3)}{8}$

8. The value(s) of x for which the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 5^n}$ converges is

(a) $-2 \le x < 8$

(b) $-2 < x \le 8$

(c) $-2 \le x \le 8$

(d) $-2 < x < 8$

(e) $-\infty < x < \infty$

Part II: Multiple Choice, Calculators Allowed

9. Find a power series representation for $f(x)=\dfrac{x^2}{1-3x}$.

(a) $x^2+x^3+x^4+x^5+\cdots$

(b) $x^2-x^3+x^4-x^5+\cdots$

(c) $1+3x+9x^2+27x^3+\cdots$

(d) $x^2+3x^3+9x^4+27x^5+\cdots$

(e) $x^2-3x^3+9x^4-27x^5+\cdots$

10. A cable that weighs 3 pounds per foot is 40 feet long and hangs from the top of a tall building. How much work is done in pulling the cable to the top of the building?

(a) 2100 $ft\cdot lbs$

(b) 2200 $ft\cdot lbs$

(c) 2300 $ft\cdot lbs$

(d) 2400 $ft\cdot lbs$

(e) 2500 $ft\cdot lbs$

11. Give the Maclaurin series expansion of $f(x)=\cos x$.

(a) $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots$

(b) $1+x+\dfrac{x^2}{2}+\dfrac{x^3}{3}+\cdots$

(c) $x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+\dfrac{x^7}{7}+\cdots$

(d) $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\cdots$

(e) $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\cdots$

Part II: Multiple Choice, Calculators Allowed

12. Suppose $20\ ft \cdot lb$ of work is required to stretch a spring from its natural length of 24 inches to a length of 36 inches. What is the spring constant k?

(a) $k = 45\ \frac{lb}{ft}$

(b) $k = 40\ \frac{lb}{ft}$

(c) $k = \frac{40}{3}\ \frac{lb}{ft}$

(d) $k = \frac{1}{3}\ \frac{lb}{ft}$

(e) $k = \frac{1}{18}\ \frac{lb}{ft}$

MATH 1242 – CALCULUS II SPRING 2022

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: Last Name

PRINT: First Name

PRINT: Student ID #

PRINT: Instructor

PRINT: Section

This exam is divided into three parts. **Calculators are allowed on Part II, but only after Part I has been handed in and the proctor has announced that calculators may be used.** You have 3 hours for the entire exam – you may hand in Part II of the exam at the end of the exam along with Part III.

PART III

- Part III consists of 5 free response problems. Each one of these problems is worth 8 points. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question you are required to show complete and detailed justification of your answer by clearly and neatly showing all your work.
- Final answers (correct or incorrect) without supporting work will not receive any credit. Conversely, incorrect final answers that are supported by correct and clearly shown processes will receive partial credit. It is important that you SHOW ALL YOUR WORK, neatly and clearly.
- Work that is illegible or disorganized will not be graded.
- All work must be shown in the space provided. You may use the blank back of each page as scratch paper, but work done on these pages or other scratch paper will not be considered.
- Please provide final answers in the final answer boxes. (Please do not include any other work or markings in the final answer boxes.)
- Make sure you print the required information (name/ID number) on EVERY page of the test booklet.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

MATH 1242
Common Final Exam
Spring 2022

Part III: Free Response
Calculators Allowed
Work must be shown to receive credit

PRINT: Last, First

1. Evaluate each integral. You must show all your work in order to receive credit. 8 POINTS

Show work in this space

a. $\int x^2\sqrt{1-x^3}\,dx$

Final Answer

b. $\int \frac{\ln x}{x^3}\,dx$

Final Answer

c. $\int \frac{x^3+3x+5}{x-1}\,dx$

Final Answer

2. Consider the finite region $\Re$ in the first quadrant between the graphs of $y = \frac{1}{x}$, $x = 2$ and $y = 2$.

In each case below,
(1) sketch the **3 dimensional solid formed** when the region is rotated about the given line
(2) set up the integral that gives the volume of the solid (using the technique indicated). 8 POINTS

Show work in this space

a. Rotate the region about the x-axis ($y = 0$). Use the disc/washer method.

Set up the integral

b. Rotate the region about the line $y = 4$. Use the disc/washer method.

Set up the integral

c. Rotate the region about the line $x = -1$. Use the shell method.

Set up the integral

PRINT: Last, First

3. Consider the function $f(x) = \ln\left(\sqrt{x}\right)$. 8 POINTS

a. Use the Trapezoidal rule with n = 4 subintervals to approximate $\int_2^{10} \ln\left(\sqrt{x}\right) dx$. Give the equation that calculates this value and then evaluate. (Round your answer to 6 decimal places).

Give the equation that calculates this value

Use your calculator to evaluate.

b. Recall that the error estimate when the Trapezoidal rule is use to approximate $\int_a^b f(x)\,dx$ is given by $|E_T| \le \frac{K(b-a)^3}{12n^2}$ where $K \ge |f''(x)|$ for $a \le x \le b$.

i. Use the given error bound formula to find the value of *K*, when the Trapezoidal rule with n = 4 subintervals is used to approximate $\int_2^{10} \ln\left(\sqrt{x}\right) dx$.

$K =$

ii. Use the given error bound formula to find an upper bound for the error, when the Trapezoidal rule with n = 4 subintervals is used to approximate $\int_2^{10} \ln\left(\sqrt{x}\right) dx$. (Round your answer to 6 decimal places).

$|E_T| \le$

iii. Find the smallest value of n that must be chosen to ensure accuracy to be within 0.001 if the Trapezoidal rule is used to approximate $\int_2^{10} \ln\left(\sqrt{x}\right) dx$.

$n =$

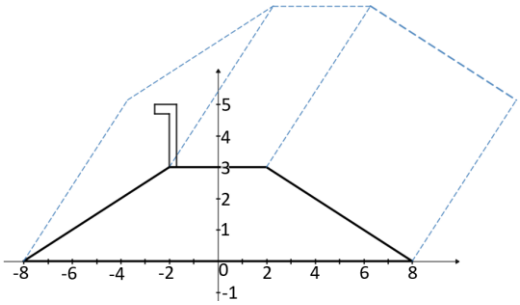
PRINT: Last, First

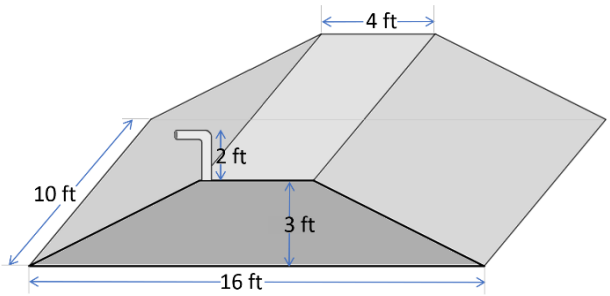
4. A large tank has the shape of a trapezoidal prism on its side (see diagram) and is filled to the top with water. The trapezoidal face of the tank has a bottom width of 16 feet and a top width of 4 feet. The tank is 3 ft tall and 10 feet long. The weight of water is 62.5 lb / ft^3.

Set up the integral that calculates the amount of work required to pump all the water out of the tank to a height of 2 feet above the top of the tank, placing the tank in the coordinate plane as shown. Then, use your calculator to evaluate. Round your answer to the nearest whole number and include the proper units.

8 POINTS

Show work in this space





Set up the integral.

Use your calculator to evaluate.

5. Consider the Maclaurin Series expansion for $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ldots$, valid for all real numbers such that $-1 \le x \le 1$. 8 POINTS

Show work in this space

a. Use the given expansion to find the first four terms of the expansion for $x^2 \tan^{-1}(x)$.

Final Answer

b. Use the expansion you found in part (a) and only the first four terms to approximate $\int_0^1 x^2 \tan^{-1}(x)\, dx$. Round your answer to 6 decimal places.

Final Answer

c. Use the Alternating Series Estimation Theorem to find an upper bound the error in your approximation in part (b). Round your answer to 6 decimal places.

Final Answer