On the dynamics of actions on compact metrizable spaces

Jade Raymond

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Abstract

In Chapter 1, we show that every shift of finite type (SFT), sofic shift, and strongly irreducible shift on locally finite groups have strong dynamical properties. We show that every sofic shift is an SFT, every SFT is strongly irreducible, and every strongly irreducible shift is an SFT, among other properties. Furthermore, we show that if any of these properties hold for a group, then the group must be locally finite. In pursuit of these results we present a formal construction of free extension shifts on a group G, which takes a shift on a subgroup H of G and naturally extends it to a shift on all of G.

In Chapter 2, we study measurable dynamical systems on compact metrizable spaces. We introduce the completion of a dynamical system, and show that it has many strong properties. We introduce Birkhoff systems, for which a version of the pointwise ergodic theorem holds as many classically systems do, using completions. For Birkhoff systems, we define dynamical independence, a property which induces probabilistic independence in ergodic measures. We apply the concepts to a variety of systems, and prove an extension of De Finetti's Theorem which characterizes exchangeability in terms of dynamical properties.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223