

SYLLABUS, Qualifying Exam, 5174

1. Linear equations of the first order with two variables (with variable coefficients)
2. Derivation of the wave equation (for a string and a rod), initial and boundary conditions.
3. Derivation of the heat equation, initial and boundary conditions.
4. Well-posed problems, examples of ill-posed problems.

Cauchy problem

1. Cauchy problem for the wave equation, $d=1$.
 - a) Unique solvability, d'Alembert formula.
 - b) Conservation of energy.
 - c) Reduction of the solutions of the Dirichlet and Neumann problems on the half line to the Cauchy problem.
 - d) Propagation and reflection of waves (Sketch a string profile at the given instants).
2. Solution of the wave equation on the half line with arbitrary boundary conditions.
3. Heat equation, $d=1$.
 - a) Solvability of the Cauchy problem, Green's function.
 - b) Maximum principle in a rectangle.
 - c) Maximal principle in a strip, uniqueness of the solution of the Cauchy problem.
 - d) Energy estimate.

Initial-boundary Problems

1. Conservation of energy, uniqueness and stability of the solutions.

2. Separation of variables. Solutions of homogeneous and inhomogeneous wave and heat equations with arbitrary boundary conditions, resonances.
3. Properties of eigenvalues and eigenvectors of symmetric positive operators
4. Sturm-Liouville problem. Properties of eigenvalues and eigenvectors.
5. Multiplicity of eigenvalues of the Sturm-Liouville problem.

Laplace Equation.

1. Relation between analytic and harmonic functions.
2. Maximum principle for the Laplace equation, uniqueness and stability of solutions of the Dirichlet problem.
3. Solutions of boundary problems in rectangles, cubes, circles, wedges, and annuli.
4. Poisson's formula.
5. Mean value theorem.
6. Differentiability of harmonic functions.
7. Green's identities.
8. Theorem on the unique solvability of the Neumann problem.
9. Mean value theorem and the maximum principle in dimension $n=3$.
10. Representation formula.
11. Fundamental solution.
12. Green's function and representation of the solutions of the Dirichlet problem.
13. Construction of the Green's function by reflections.