

PARKING TREES AND THE TORIC g -VECTOR OF NESTOHEDRA

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Abstract

We express the toric g -vector entries of any simple polytope as a nonnegative integer linear combination of its γ -vector entries. Using this expression we obtain that the toric g -vector of the associahedron is the ascent statistic of 123-avoiding parking functions. An analogous result holds for the cyclohedron and 123-avoiding functions. We prove that the toric g -vector of the permutahedron records the ascent statistics of parking trees representing 123-avoiding parking functions. We indicate how our approach extends to all chordal nestohedra.

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