Discrete spectrum in gaps of the continuous spectrum for finite difference operators on the lattice and the graphene operator

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Abstract

The object of study of this dissertation is the distribution of discrete eigenvalues of operators. Specifically, we investigate the number and location of discrete eigenvalues within the gaps of the continuous spectrum of three operators, two discrete and onecontinuous. The discrete operators are defined on the lattice $\mathbb{Z}^d$ and have a potential V. However, one has a positive coupling constant and the other does not. For the first operator, we compute the asymptotics of the number of eigenvalues passing through a fixed point inside a spectral gap when the coupling constant increases from 0 to infinity. For the second operator, we estimate the number of discrete eigenvalues. This estimate is a generalization of the CLR inequality in dimension $d \geq 3$. The continuous operator is the graphene operator acting on the Sobolev space $H^2(\mathbb{R}^2, \mathbb{C}^2)$. This operator describes the two-layer graphene structure. The potential V is multiplied by a coupling constant α. We prove that the eigenvalue asymptotics depends on the integrability of V. Particularly, if V is not integrable, then the asymptotics is determined by the rate of decay of V at infinity. If V is integrable, then it is determined by the integral of V

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