MATH 1241 – CALCULUS I

SPRING 2023

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: FIRST NAME	LAST NAME

PRINT: STUDENT ID NUMBER	PRINT: INSTRUCTOR	PRINT: SECTION

This exam is divided into three parts. **Calculators are not allowed on Part I and during the first hour of the exam.** You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the exam during the first hour, but you cannot use your calculator during this time. You may use your calculator only after you have submitted Part 1 and the exam proctor has announced that calculators are allowed.

PART I

- Part I consists of 17 multiple choice problems. These problems must be answered without the use of a calculator.
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer make sure that you completely erase your old answer and any extraneous marks. You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "A" to indicate Part I.
- You must hand in the test booklet and bubble sheet for Part I exactly one hours after the exam started.
- Only scratch paper provided by the proctor may be used.

Part I: Multiple Choice, No Calculators

1. Give the derivative for $f(x)=3x^4-6x+2$.

   (a) $f'(x)=7x-6$

   (b) $f'(x)=7x^3-6x+2$

   (c) $f'(x)=3x-6$

   (d) $f'(x)=12x^3-6$

   (e) $f'(x)=12x+2$

2. Let $g(x)=\sqrt{\sin x}$. Find $g'(x)$.

   (a) $\sqrt{\cos x}$

   (b) $\frac{1}{2\sqrt{\sin x}}$

   (c) $\sqrt{\sin x}\cdot\cos x$

   (d) $-\frac{\cos x}{2\sqrt{\sin x}}$

   (e) $\frac{\cos x}{2\sqrt{\sin x}}$

3. Evaluate $\lim\limits_{x\to 8}\frac{x^2-8x}{x-8}$.

   (a) 0

   (b) 8

   (c) 12

   (d) 16

   (e) The limit does not exist

4. Find the equation of the tangent line to the graph of $g(x)=(x^2+1)\cdot\ln x$ at the point where $x=1$.

   (a) $y=2x-2$

   (b) $y=2x+2$

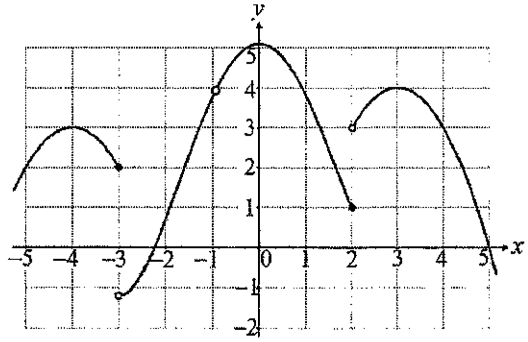
   (c) $y=2x-1$

   (d) $y=2x+1$

   (e) $y=-2x+1$

Part I: Multiple Choice, No Calculators

5. The graph of $f(x)$ is shown below. Give $\lim_{x \to -3^-} f(x)$.



(a) 3

(b) 2

(c) 1

(d) −1

(e) The limit does not exist

6. Let $f(x)=2x^4-24x+18$. Evaluate $\lim_{h \to 0} \dfrac{f(2+h)-f(2)}{h}$.

(a) -20

(b) 22

(c) 40

(d) 52

(e) 74

7. Evaluate $\lim_{x \to \infty} \sqrt{\dfrac{2x+27x^3}{3x^3-x+1}}$.

(a) $\sqrt{\frac{2}{3}}$

(b) $\frac{2}{3}$

(c) 3

(d) 9

(e) The limit does not exist

8. Calculate the derivative of $h(x)=x^5 \cdot 5^x$

(a) $h'(x)=5x^4 \cdot 5^x + x^5 \cdot \frac{5^x}{\ln 5}$

(b) $h'(x)=5x^4 \cdot 5^x + x^5 \cdot 5^x \cdot \ln 5$

(c) $h'(x)=5x^4 \cdot 5^x + x^5 \cdot x \cdot 5^{x-1}$

(d) $h'(x)=5x^4 \cdot 5^x \cdot \ln 5$

(e) $h'(x)=5x^4 \cdot \frac{5^x}{\ln 5}$

Part I: Multiple Choice, No Calculators

9. Calculate the derivative of $f(x)=\ln\left(x^3+x\right)$

(a) $f'(x)=\dfrac{3x^2+1}{x^3+x}$

(b) $f'(x)=\dfrac{1}{x^3+x}$

(c) $f'(x)=\dfrac{1}{3x^2+1}$

(d) $f'(x)=\ln\left(3x^2+1\right)$

(e) $f'(x)=\ln\left(x^3+x\right)\left(3x^2+1\right)$

10. Calculate the derivative of $f(x)=\tan\left(\dfrac{x^2}{2x+10}\right)$

(a) $f'(x)=\tan\left(\dfrac{2x^2+20x}{(2x+10)^2}\right)$

(b) $f'(x)=\sec^2\left(\dfrac{2x^2+20x}{(2x+10)^2}\right)$

(c) $f'(x)=\sec\left(\dfrac{x^2}{2x+10}\right)\cdot\tan\left(\dfrac{x^2}{2x+10}\right)\cdot\dfrac{2x^2+20x}{(2x+10)^2}$

(d) $f'(x)=\sec\left(\dfrac{2x^2+20x}{(2x+10)^2}\right)\cdot\tan\left(\dfrac{2x^2+20x}{(2x+10)^2}\right)$

(e) $f'(x)=\sec^2\left(\dfrac{x^2}{2x+10}\right)\cdot\dfrac{2x^2+20x}{(2x+10)^2}$

11. The derivative of the function $f(x)=\dfrac{x^4+10x^2-4}{x^2}$ is

(a) $2x-\frac{8}{x^3}$

(b) $2x+\frac{8}{x^3}$

(c) $4x^3+10$

(d) $\frac{4x^3+20x}{x^4}$

(e) $\frac{4x^3+20x}{2x}$

Part I: Multiple Choice, No Calculators

12. Suppose f^{-1} is the inverse function of a differentiable function f, $f(3)=10$ and $f'(3)=\frac{1}{4}$.

Then $\left(f^{-1}\right)'(10)=$

(a) $\frac{1}{10}$

(b) $\frac{1}{4}$

(c) 3

(d) 4

(e) 10

13. The derivative of the function $f(x)=\arcsin\left(\frac{1}{x}\right)$ is

(a) $f'(x)=-\dfrac{\arccos\left(\frac{1}{x}\right)}{x^2}$

(b) $f'(x)=-\dfrac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$

(c) $f'(x)=\dfrac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$

(d) $f'(x)=-\dfrac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$

(e) $f'(x)=\dfrac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$

14. The graph for a function $f(x)$ is shown. Which of the following must be true?

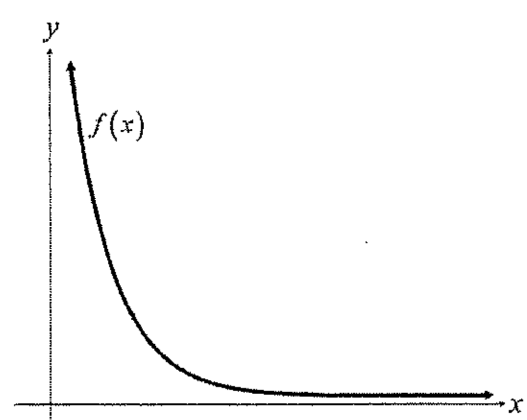
(a) $f(x)>0$, $f'(x)<0$ and $f''(x)>0$ for all $x>0$.

(b) $f(x)>0$, $f'(x)>0$ and $f''(x)>0$ for all $x>0$.

(c) $f(x)>0$, $f'(x)<0$ and $f''(x)<0$ for all $x>0$.

(d) $f(x)>0$, $f'(x)>0$ and $f''(x)<0$ for all $x>0$.

(e) $f(x)<0$, $f'(x)<0$ and $f''(x)<0$ for all $x>0$.



Part I: Multiple Choice, No Calculators

15. If the function $f(x)=kx^3-6x^2-10x+4$ has an inflection point at $x=1$, then k is equal to

(a) -4

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) 2

(e) 4

16. Let $y=\dfrac{e^{3x}\sqrt{5x^2-2}}{(x+1)^2}$. Use logarithmic differentiation to find $f'(x)$.

(a) $\dfrac{dy}{dx}=\dfrac{e^{3x}\sqrt{5x^2-2}}{(x+1)^2}\cdot\left(3e^{3x}+\dfrac{5x}{5x^2-2}-\dfrac{2}{x+1}\right)$

(b) $\dfrac{dy}{dx}=\dfrac{e^{3x}\sqrt{5x^2-2}}{(x+1)^2}\cdot\left(3e^{3x}+\dfrac{1}{10x^2-4}-\dfrac{1}{x+1}\right)$

(c) $\dfrac{dy}{dx}=\dfrac{e^{3x}\sqrt{5x^2-2}}{(x+1)^2}\cdot\left(3+\dfrac{5x}{5x^2-2}-\dfrac{2}{x+1}\right)$

(d) $\dfrac{dy}{dx}=3e^{3x}\cdot\dfrac{5x}{\sqrt{5x^2-2}}\cdot\dfrac{1}{2(x+1)}$

(e) $\dfrac{dy}{dx}=3e^{3x}\cdot\dfrac{5x}{\sqrt{5x^2-2}}\cdot2(x+1)$

17. Find the general antiderivative of $g(x)=18x^2-\dfrac{8}{x^3}+\dfrac{1}{1+x^2}$.

(a) $6x^3-\dfrac{4}{x^2}+\ln\left|1+x^2\right|+C$

(b) $6x^3+\dfrac{4}{x^2}+\ln\left|1+x^2\right|+C$

(c) $6x^3+\dfrac{16}{x^4}+\arctan x+C$

(d) $6x^3-\dfrac{4}{x^2}+\arctan x+C$

(e) $6x^3+\dfrac{4}{x^2}+\arctan x+C$

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PART II

- Part II consists of 14 multiple choice problems. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
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- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "B" to indicate Part II.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used

Part II: Multiple Choice, Calculators Allowed

1. Let $f(x)=\frac{5}{2}x^2-\frac{1}{x}$. Evaluate $f'(1)$.

   (a) -3

   (b) 0

   (c) 3

   (d) 6

   (e) 9

2. Evaluate $\lim_{x\to 7}\frac{\sqrt{x+18}-5}{x-7}$.

   (a) $\frac{1}{10}$

   (b) $\frac{16}{9}$

   (c) $\frac{1}{9}$

   (d) 1

   (e) Does Not Exist

3. A particle is moving with acceleration $a(t)=30t+2$ where t is measured in seconds and $a(t)$ in feet per second squared. Its position at time $t=0$ is $s(0)=13$ feet and its velocity at time $t=0$ is $v(0)=17$ feet per second. What is its position at time $t=3$ seconds?

   (a) -24 feet

   (b) 18 feet

   (c) 128 feet

   (d) 208 feet

   (e) 288 feet

Part II: Multiple Choice, Calculators Allowed

4. Let $f(x)=\begin{cases} cx^2-15x-13 & , x<4 \\ \sqrt{10x+9} & , x\geq 4 \end{cases}$. For what value of c is f continuous?

(a) 2

(b) 3

(c) 4

(d) 5

(e) 6

5. Suppose f is a continuous function, so that $f(5)=5$, $f'(5)=0$, $f''(5)=2$. Which of the following is true about the graph of the function f?

(a) f has an inflection point at $x=5$

(b) f has a local minimum value at $x=5$

(c) f has a local maximum value at $x=5$

(d) f has increasing slope at $x=5$

(e) f has decreasing slope at $x=5$

6. Consider the function $f(x)=3x^2+6x+5$ on the interval $-5\leq x\leq 5$. The absolute minimum and maximum values of the function are

(a) Absolute minimum $=2$ and Absolute maximum $=50$

(b) Absolute minimum $=2$ and Absolute maximum $=110$

(c) Absolute minimum $=50$ and Absolute maximum $=110$

(d) Absolute minimum $=14$ and Absolute maximum $=110$

(e) Absolute minimum $=14$ and Absolute maximum $=50$

Part II: Multiple Choice, Calculators Allowed

7. Find the linear approximation for the function $f(x)=\sqrt{x}$ at $a=16$. Then use the approximation formula to estimate $\sqrt{15.1}$.

(a) 3.88475

(b) 3.88508

(c) 3.88587

(d) 3.88623

(e) 3.88750

8. The area of a disc (circle) is given by $A=\pi r^2$. Use the differential dA to approximate the change in area when the radius increases from $r=5$ to $r=5.2$.

(a) 6.283

(b) 6.303

(c) 6.341

(d) 6.384

(e) 6.409

9. The **first derivative** of a function f is given by $f'(x)=(x-2)^2(x-4)(x+2)$. Which one of the following statements is correct?

(a) f has a relative maximum at $x=4$

(b) f is increasing on the interval $(2,4)$

(c) f has a relative maximum at $x=-2$

(d) f has a relative minimum at $x=-2$

(e) f is decreasing on the interval $(-\infty,-2)$

Part II: Multiple Choice, Calculators Allowed

10. Suppose that $5x + xy^3 = 4y - 2$. Find the derivative $\frac{dy}{dx}$ at the point $(1, -1)$.

(a) -4

(b) $-\frac{5}{4}$

(c) $\frac{1}{4}$

(d) $\frac{5}{4}$

(e) 4

11. Find all of the critical numbers for the function $f(x) = x + 2\cos x$ on the interval $[0, 2\pi]$.

(a) $x = \frac{\pi}{6}$

(b) $x = \frac{5\pi}{6}$

(c) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

(d) $x = 0$ and $x = \frac{\pi}{6}$

(e) $x = 0$ and $x = \frac{5\pi}{6}$

12. The function $f(x) = x^4 - 2x - 1$ has one root on the interval $(1, 2)$. Using Newton's method, with $x_1 = 1.5$ as your initial approximation, find x_2, the next approximation. (You are not being asked for the exact solution.) Round your answer to three decimal places.

(a) 1.408

(b) 1.417

(c) 1.429

(d) 1.441

(e) 1.448

Part II: Multiple Choice, Calculators Allowed

13. Suppose the derivative of a function $f(x)$ satisfy $2 \le f'(x) \le 8$ for $1 \le x \le 3$. By the Mean Value Theorem, we can conclude that the difference $f(3)-f(1)$ is between

(a) 1 and 3

(b) 2 and 8

(c) 4 and 8

(d) 4 and 16

(e) 8 and 16

14. The graph of $f'(x)$ is shown below. Give the all interval(s) where the function $f(x)$ is concave up.

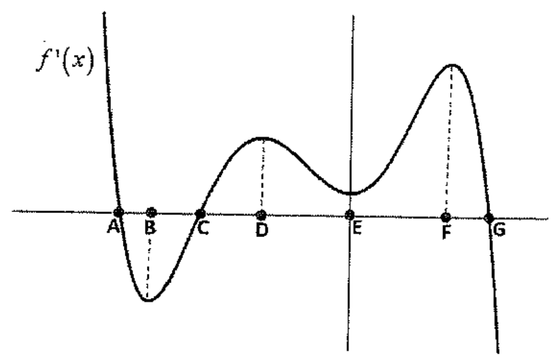
(a) $(-\infty, B) \cup (D, E) \cup (F, \infty)$

(b) $(B, D) \cup (E, F)$

(c) $(-\infty, A) \cup (F, \infty)$

(d) $(A, C) \cup (G, \infty)$

(e) $(-\infty, A) \cup (C, G)$



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This exam is divided into three parts. **Calculators are allowed on Part II, but only after Part I has been handed in and the proctor has announced that calculators may be used.** You have 3 hours for the entire exam – you may hand in Part II of the exam at the end of the exam along with Part III.

PART III

- Part III consists of 5 free response problems. Each one of these problems is worth 8 points. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI Nspire, TI 89, etc. may not be used.)
- For each question you are required to show complete and detailed justification of your answer by clearly and neatly showing all your work.
- Final answers (correct or incorrect) without supporting work will not receive any credit. Conversely, incorrect final answers that are supported by correct and clearly shown processes will receive partial credit. It is important that you SHOW ALL YOUR WORK, neatly and clearly.
- Work that is illegible or disorganized will not be graded. Please use a sharp #2 (or darker) pencil or black ink.
- All work must be shown in the space provided. You may use the blank back of each page as scratch paper, but work done on these pages or other scratch paper will not be considered.
- Please provide final answers in the final answer boxes. (Please do not include any other work or markings in the final answer boxes.)
- Make sure you print the required information (name/ID number) on EVERY page of the test booklet.
- At the end of the exam you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

Part III: Free Response
Calculators Allowed
Work must be shown to receive credit

PRINT: Last, First

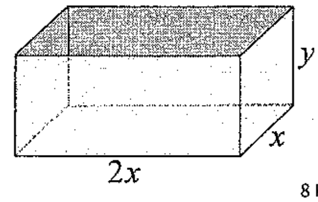
1. Calculate the following limits using L'Hopitals rule. Show every step of your work. Leave all answers as exact values.

a. $$\lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}$$

b. $$\lim_{x \to 0^+} x^2 \ln x$$

c. $$\lim_{x \to 1^+} x^{\frac{1}{(1-x)}} =$$

2. 2,400 square inches of material will be used to make a rectangular storage container with an open top. The length of the base, is twice the width. Find the largest possible volume of the box. Show each step of your work in order to receive credit.



8 POINTS

a. Consider the expression for the surface area of the container in terms of x and y. Using the fact that the surface area must be exactly 2,400 square inches, express y in terms of x.

$y =$

b. Express the volume of the container in terms of x and y.

$Volume =$

c. Using the results in part (a), express the volume of the container as a function of x.

$V(x) =$

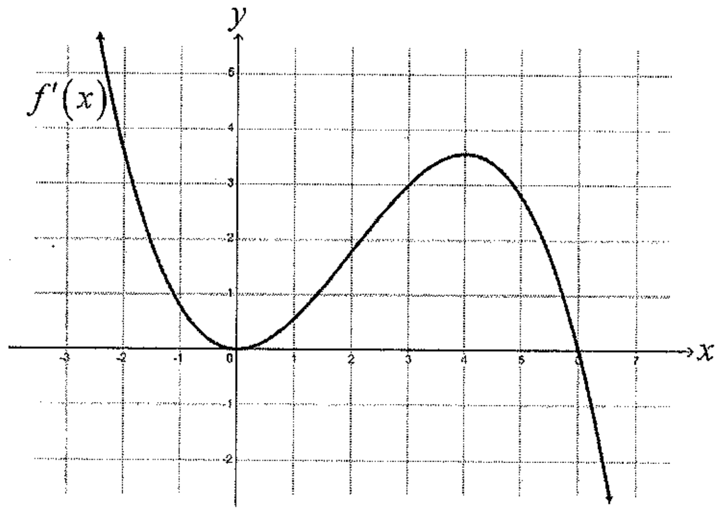
d. Finally, find maximum volume of the container. (Round answer to the nearest whole number and include units).

$Volume =$

Part III: Free Response
Calculators Allowed
Work must be shown to receive credit

PRINT: Last, First

3. The graph of $y = f'(x)$ is shown below. Answer the following questions about the original function $f(x)$. 8 POINTS



a. Give all of the intervals where $f(x)$ is strictly increasing.

b. Give all of the intervals where $f(x)$ is concave up.

c. Give the all the x coordinates where $f(x)$ has local maximum(s). List "none", if there are none.

d. Give the all the x coordinates where $f(x)$ has local minimum(s). List "none", if there are none.

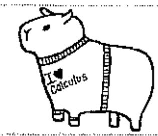
e. Give the all the x coordinates where $f(x)$ has inflection points. List "none", if there are none.

Part III: Free Response
Calculators Allowed
Work must be shown to receive credit

PRINT: Last, First

8 POINTS

4. The capybara is a giant cavy rodent native to South America.



a. A study found that the population of capybaras in a region of Venezuela has increased form 10,000 in 2015 to 12,500 in 2020. Find the growth constant k, and give an expression $P(t)$ for the number of capybaras as a function of t. Round your k value to 3 decimal places.

$k =$

$P(t) =$

b. A different study tracked the population of in Peru, where there were 6000 capybaras in 2015. The population growth constant in this region was found to be k = 0.075.

i. How many capybaras are there in 2017? Round to the nearest whole number.

ii. At what rate are is the capybara population increasing in 2017? Round to the nearest whole number.

iii. If this rate of growth continues, after how long will the capybara population reach 20,000? Round your answer to the nearest year.

5. Suppose that we have two resistors connected in parallel with resistances $R_1 = A$ and $R_2 = B$ measured in ohms (Ω). The total resistance, R is then given by $\frac{1}{R} = \frac{1}{A} + \frac{1}{B}$. If A is decreasing at a rate of 2 Ω/min and B is increasing at a rate of 8 Ω/min, at what rate is R changing when $A = 70\ \Omega$ and $B = 95\ \Omega$?

a. Find a formula for the rate of change $\frac{dR}{dt}$ of the resistance as a function of R, A, B, $\frac{dA}{dt}$, and $\frac{dB}{dt}$.

b. Calculate the rate of change $\frac{dR}{dt}$. (Round answer to 3 decimal places and include units).