MATH 1241 – CALCULUS I Spring 2024

COMMON FINAL EXAMINATION

UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
MATHEMATICS & STATISTICS

PRINT: FIRST NAME	LAST NAME

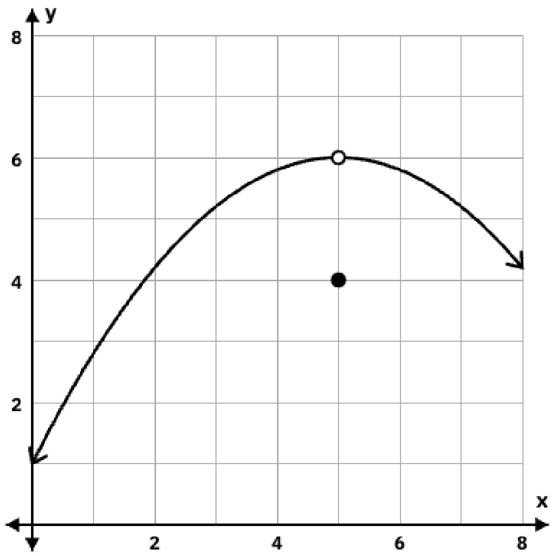
PRINT: STUDENT ID NUMBER	PRINT: INSTRUCTOR	PRINT: SECTION

This exam is divided into three parts. **Calculators are not allowed on Part I and during the first hour of the exam**. You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the exam during the first hour, but you cannot use your calculator during this time. You may use your calculator only after you have submitted Part 1 and the exam proctor has announced that calculators are allowed.

PART I

- Part I consists of 15 multiple choice problems. These problems must be answered without the use of a calculator.
- For each question choose the response which best fits the question. You must indicate each answer on the provided bubble sheet by completely shading the bubble with a dark pencil.
- If you wish to change your answer, ***do not erase the mark***! Place an X over the answer you want to erase and fill-in the correct bubble.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the bubble sheet.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- There is no penalty for guessing.
- Make sure your clearly print your name and student ID # on the test booklets and bubble sheet.
- In the "version" field of the bubble sheet, bubble "B" to indicate Part II.
- At the end of the exam, you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

Part 1 - No calculators



1.1 The graph of $f(x)$ is shown above. Find $\lim_{x \to 5} f(x)$

(a) 6

(b) 4

(c) 0

(d) 5

(e) The limit does not exist (DNE).

1.2 Find the derivative of $g(x) = 3x^4 - 6x + 2$

(a) $7x - 6$

(b) $12x - 6$

(c) $7x^3 - 6x$

(d) $7x^3 - 6$

(e) $12x^3 - 6$

1.3 Find $\lim_{x \to 1} \dfrac{x^2 + 8x - 9}{x^2 - 1}$

(a) 10

(b) 1

(c) 0

(d) 5

(e) The limit does not exist (DNE).

1.4 Is $f(x) = \begin{cases} x^3 - 2x & \text{if } x < 2 \\ x + 1 & \text{if } x \geq 2 \end{cases}$ continuous at $x = 2$? Why?

(a) No, because $f(2)$ is undefined.

(b) Yes, because $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

(c) No, because $\lim_{x \to 2} f(x)$ exists, but $\lim_{x \to 2} f(x) \neq f(2)$

(d) No, because $\lim_{x \to 2} f(x)$ does not exist.

(e) Yes, because $\lim_{x \to 2} f(x) = f(2)$

1.5 Suppose that $y = -2x + 5$ is the equation of the tangent line to a function $f(x)$ at $(3, f(3))$. If possible, find $f'(3)$.

(a) -1

(b) 1

(c) -2

(d) 0

(e) 2

1.6 Which of the following is the derivative of $f(t) = \sqrt[3]{t} + \dfrac{1}{\sqrt[3]{t^2}}$?

(a) $\dfrac{1}{3}t^{-2/3} - \dfrac{2}{3}t^{-5/3}$

(b) $\dfrac{1}{3}t^{-2/3} - \dfrac{2}{3}t^{1/3}$

(c) $\dfrac{1}{3}t - \dfrac{2}{3t}$

(d) $\dfrac{1}{3}t^{-2/3} + \dfrac{2}{3}t^{-5/3}$

(e) $\dfrac{1}{3}t^{2/3} - \dfrac{2}{3}t^{5/3}$

1.7 Find the derivative of $f(x) = \dfrac{x^2 + 1}{x^3 - 1}$. Do not simplify.

(a) $f'(x) = \dfrac{2x + 1}{3x^2 - 1}$

(b) $f'(x) = \dfrac{(x^3 - 1)(2x) - (x^2 + 1)(3x^2)}{(x^3 - 1)^2}$

(c) $f'(x) = \dfrac{(x^2 + 1)(3x^2) + (x^3 - 1)(2x)}{(x^3 - 1)^2}$

(d) $f'(x) = \dfrac{2x}{3x^2 - 1}$

(e) $f'(x) = \dfrac{(x^2 + 1)(3x^2) - (x^3 - 1)(2x)}{(x^3 - 1)^2}$

1.8 $f(x) = x^2 \cdot \sin(x^4+1)$. Find $f'(x)$.

(a) $2x \cdot \cos\left(x^4+1\right) \cdot \left(4x^3\right)$

(b) $2x \cdot \sin\left(x^4+1\right) - (x^2)\cos\left(x^4+1\right) \cdot \left(4x^3\right)$

(c) $2x \cdot \cos\left(x^4+1\right)$

(d) $2x \cdot \cos\left(x^4+1\right) - (x^2)\sin\left(x^4+1\right)$

(e) $2x \cdot \sin\left(x^4+1\right) + (x^2)\cos\left(x^4+1\right) \cdot \left(4x^3\right)$

1.9 Find $\dfrac{dy}{dx}$ for $x^2 + 5xy + y^3 = -7$ at the point $(2, -1)$.

(a) $-\dfrac{7}{5}$

(b) $\dfrac{1}{13}$

(c) $-\dfrac{8}{7}$

(d) $\dfrac{1}{7}$

(e) $-\dfrac{3}{4}$

1.10 Find the derivative of $f(x) = \ln\left(6x^3 + 5x^2 + 2\right)$

(a) $f'(x) = \dfrac{18x^2 + 10x}{6x^3 + 5x^2 + 2}$

(b) $f'(x) = \dfrac{\ln(18x^2 + 10x)}{6x^3 + 5x^2 + 2}$

(c) $f'(x) = \dfrac{36x + 10}{18x^2 + 10x}$

(d) $f'(x) = \dfrac{18x^2 + 10x}{\ln(6x^3 + 5x^2 + 2)}$

(e) $f'(x) = \dfrac{1}{18x^2 + 10x}$

1.11 Find the derivative of $f(x) = \sin^{-1}\left(4x^3 - 8x\right)$

(a) $f'(x) = \cos^{-1}\left(4x^3 - 8x\right) \cdot \left(12x^2 - 8\right)$

(b) $f'(x) = \dfrac{12x^2 - 8}{\sqrt{1 - (4x^3 - 8x)}}$

(c) $f'(x) = \cos^{-1}\left(4x^3 - 8x\right)$

(d) $f'(x) = \dfrac{12x^2 - 8}{\sqrt{1 - (4x^3 - 8x)^2}}$

(e) $f'(x) = \dfrac{4x^3 - 8x}{\sqrt{1 - (12x^2 - 8)^2}}$

1.12 Find $\lim_{x\to\infty} \dfrac{2x + 27x^3}{3x^3 - x - 1}$

(a) 9

(b) 3

(c) $\dfrac{2}{3}$

(d) -3

(e) The limit does not exist (DNE).

1.13 Find $\lim_{x\to 0} \dfrac{e^x - x - 1}{\cos x - 1}$

(a) 1

(b) -1

(c) 0

(d) ∞

(e) $-\infty$

1.14 Find the general antiderivative $f(x)$ when $f'(x) = \dfrac{5}{x^2}$

(a) $f(x) = \dfrac{5x}{\frac{x^3}{3}} + C$

(b) $f(x) = \dfrac{5}{x} + C$

(c) $f(x) = -\dfrac{5}{x} + C$

(d) $f(x) = -5\ln|x^2| + C$

(e) $f(x) = 5\ln|x^2| + C$

1.15 Find $\int (x^2 + 2)^2\, dx$.

(a) $\dfrac{x^5}{5} + 4x + C$

(b) $x^4 + 4 + C$

(c) $4x(x^2 + 2) + C$

(d) $\dfrac{(x^2 + 2)^3}{3} + C$

(e) $\dfrac{x^5}{5} + \dfrac{4x^3}{3} + 4x + C$

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PART II

- Part II consists of 13 multiple choice problems. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI-Nspire, TI 89, etc. may not be used.)
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Part 2 - Calculators allowed

2.1 Assuming $\lim_{x\to 4} h(x) = -5$, what is $\lim_{x\to 4} \dfrac{x \cdot h(x)}{x^2+4}$?

(a) $\dfrac{9}{8}$

(b) $-\dfrac{9}{8}$

(c) 1

(d) -1

(e) $-\dfrac{5}{4}$

2.2 $g(x) = \begin{cases} x^2+1 & \text{if } x < 3 \\ 2x+3 & \text{if } 3 \le x < 4 \\ -2x^2 & \text{if } x \ge 4 \end{cases}$. Find $\lim_{x\to 3^-} g(x)$.

(a) 10

(b) 9

(c) -32

(d) 11

(e) -18

2.3 $f(x) = e^{2x}$. What does $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$ represent?

(a) $f(2)$

(b) $f(4)$

(c) $f'(0)$

(d) $f'(2)$

(e) $f'(4)$

2.4 For what value(s) of k is $g(x) = \begin{cases} kx^2+3x-1 & , x \le 1 \\ \frac{k}{2}-5x & , x > 1 \end{cases}$ continuous at $x = 1$?

(a) 0

(b) 1

(c) $\dfrac{-8 \pm \sqrt{58}}{2}$

(d) -14

(e) $-4 \pm \sqrt{15}$

2.5 Suppose that for some unknown function $u(x)$, it is known that $u(2+h) - u(2) = 3h^2 + 5h$. Use the definition of the derivative to find $u'(2)$.

(a) 3

(b) 22

(c) 0

(d) 11

(e) 5

2.6 A 10 ft ladder is leaning against a wall and the top of the ladder is sliding down at a rate of 4 ft/sec (see picture below). At what rate is the distance of bottom of the ladder from the wall changing when the bottom is 6 ft from the wall?

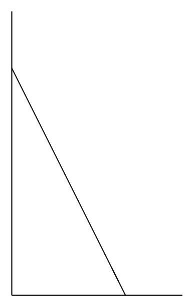
(a) $-\frac{41}{3}$ ft/sec.

(b) $\frac{16}{3}$ ft/sec.

(c) $-\frac{5}{3}$ ft/sec.

(d) $-\frac{16}{3}$ ft/sec.

(e) $\frac{41}{3}$ ft/sec.



2.7 Find the linearization, $L(x)$, of $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate $\sqrt{.75}$ to 3 decimal places.

(a) 0.875

(b) 0.625

(c) 0.866

(d) 0.825

(e) 0.884

2.8 Under which of the following conditions must $f(x)$ have a local minimum at $x = 9$?

(a) $f'(9) = 0,\ f''(9) < 0$

(b) $f'(9) > 0,\ f''(9) > 0$

(c) $f'(9) < 0,\ f''(9) < 0$

(d) $f'(9) = 0,\ f''(9) > 0$

(e) $f'(9) = 0,\ f''(9) = 0$

Use the following table for problems 2.9 and 2.10

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	2	−2	2.5	−1
2.5	−1	−3	1	2
5	6	7	−4	−3

2.9 $H(x) = f(x) \cdot g(x)$. Find $H'(2)$.

(a) −7

(b) 3

(c) 5

(d) −5

(e) −3

2.10 $K(x) = f(g(x))$. Find $K'(2)$.

(a) −3

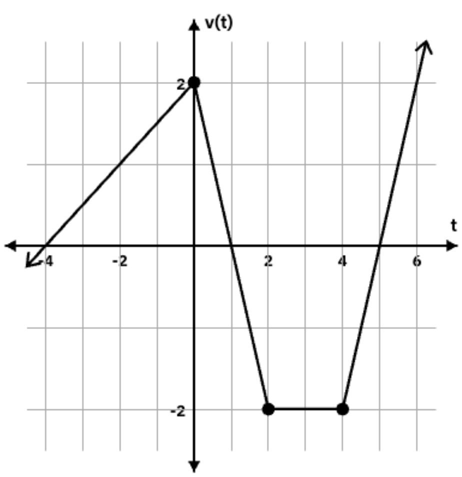
(b) −7

(c) 3

(d) 5

(e) −5

2.11 The figure below shows the velocity, $v(t)$, of a particle moving on a horizontal coordinate line.



For what values of t is the particle moving in a positive direction?

(a) $(0, \infty)$

(b) $(-\infty, 0) \cup (4, \infty)$

(c) $(0, 2)$

(d) $(-4, 1) \cup (5, \infty)$

(e) $(2, 4)$

2.12 An object is moving with acceleration $a(t) = 24t + 4$. Its position at time $t = 0$ is $s(0) = 10$ and its velocity at time $t = 0$ is $v(0) = 2$. Give the position function, $s(t)$, for the object.

(a) $s(t) = 4t^3 + 2t^2 + 2t + 2$

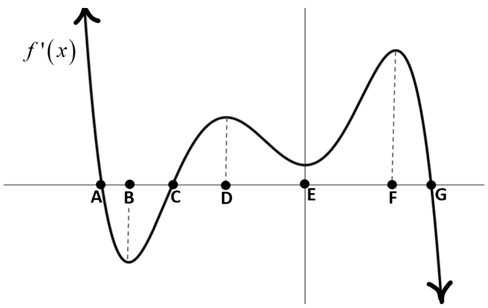
(b) $s(t) = 2t^3 + 4t^2 + 2t + 2$

(c) $s(t) = 2t^3 + 2t^2 + 4t + 2$

(d) $s(t) = 2t^3 + 4t^2 + 2t + 10$

(e) $s(t) = 4t^3 + 2t^2 + 2t + 10$

2.13 The graph of $f'(x)$ is shown below. Which of the following gives all of the intervals where the function $f(x)$ is **concave down**?



(a) $(B, C) \cup (G, \infty)$

(b) $(A, B) \cup (C, D) \cup (F, \infty)$

(c) $(-\infty, B) \cup (D, E) \cup (F, \infty)$

(d) $(A, C) \cup (G, \infty)$

(e) $(A, B) \cup (G, \infty)$

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This exam is divided into three parts. **Calculators are allowed on Part III, but only after Part I has been handed in and the proctor has announced that calculators may be used.** You have 3 hours for the entire exam – you will hand in Part III of the exam at the end of the exam along with Part III.

PART III

- Part III consists of 5 free response problems. Each one of these problems is worth 8 points. After your proctor announces that calculators may be used, you may use your calculator on this part of the exam. Texas Instruments 83 or 84 or (pre-approved) equivalent models of other brands may be used. (Note that TI-Nspire, TI 89, etc. may not be used.)
- For each question you are required to show complete and detailed justification of your answer by clearly and neatly showing all your work.
- Final answers (correct or incorrect) without supporting work will not receive any credit. Conversely, incorrect final answers that are supported by correct and clearly shown processes will receive partial credit. It is important that you SHOW ALL YOUR WORK, neatly and clearly.
- Work that is illegible or disorganized will not be graded. Please use a sharp #2 (or darker) pencil or black ink.
- All work must be shown in the space provided. You may use the blank back of each page as scratch paper, but work done on these pages or other scratch paper will not be considered.
- Please box your final answer.
- Make sure you print the required information (name/ID number) on EVERY page of the test booklet.
- At the end of the exam, you must hand in all test materials including the test booklets, bubble sheets and scratch paper.
- Only scratch paper provided by the proctor may be used.

Part 3 - Free Response, calculators allowed

3.1 It is known that a function $f(x)$ has the following properties:

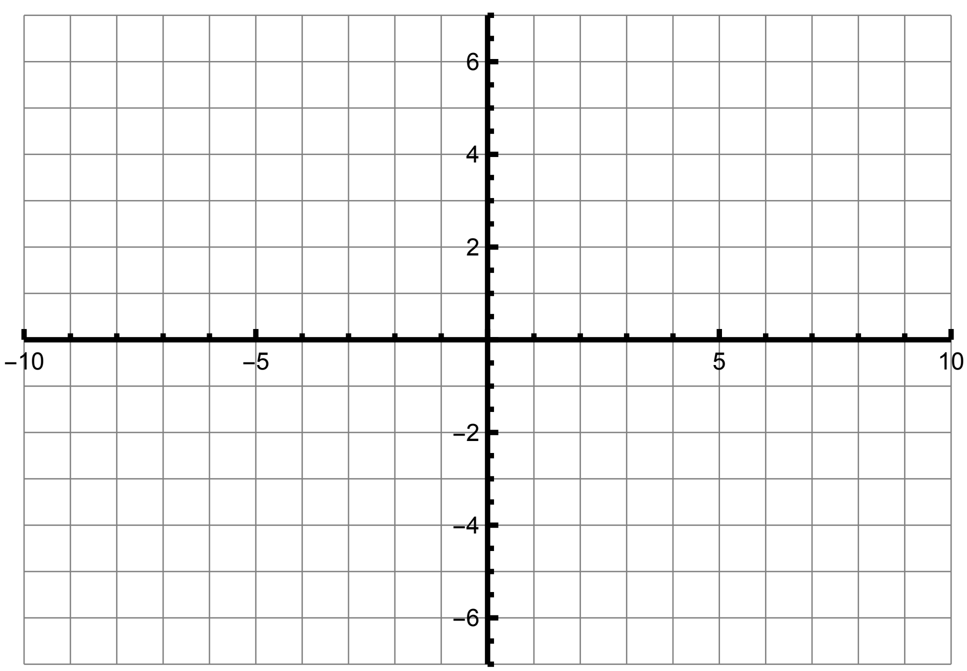
$$f(-2) = 2 \qquad f'(x) > 0 \text{ if } x < 0$$
$$f(1) = 0 \qquad f'(x) < 0 \text{ if } x > 0$$
$$\lim_{x\to\infty} f(x) = -3 \qquad f''(x) > 0 \text{ if } x < 0$$
$$\lim_{x\to-\infty} f(x) = 1 \qquad f''(x) > 0 \text{ if } x > 0$$
$$\lim_{x\to 0} f(x) = \infty$$

(a) Identify the points of discontinuity of $f(x)$.

(b) Does $f(x)$ have any vertical asymptotes? If so, what are they?

(c) Does $f(x)$ have any horizontal asymptotes? If so, what are they?

(d) Sketch the graph of $y = f(x)$



3.2 $f(x) = x^2 - x + 2$

(a) Using the definition of a derivative, find $f'(x)$

(b) Using the result from part (a), find the slope of the tangent line to $f(x) = x^2 - x + 2$ at $x = -1$.

(c) Find the equation of the tangent line at $(-1, f(-1))$. Please write your answer in slope-intercept $(y = mx + b)$ form.

3.3 $f(x) = x^3 + x - 1$ for $0 \leq x \leq 1$

(a) Evaluate $f(0)$ and $f(1)$.

(b) Noting that $f(0)$ and $f(1)$ have different signs, which theorem from Calculus ***guarantees*** there will be a root of $f(x)$ in the interval $(0, 1)$?

(c) Using Newton's Method with the first (initial) approximation $x_1 = 1$, find the second approximation, x_2, of the root, to 3 decimal places.

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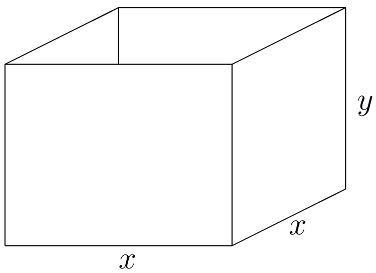
3.4 A box with a square base length x and an open top must have surface area (the area of the 4 sides plus the area of the bottom) of 147 square inches. What are the dimensions of the box, x and y, that maximize the volume? (Hint: Maximize the volume of the box, which is length·width·height.)



$x =$ $y =$

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3.5 $f(x) = (2+x)^{\frac{1}{x}}$. Using logarithmic differentiation, find $f'(2)$.

$f'(2) =$